Blade/Casing Contacts in Turbomachinery: State of the Art and Recent Developments

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Analysis

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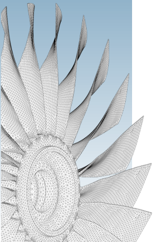
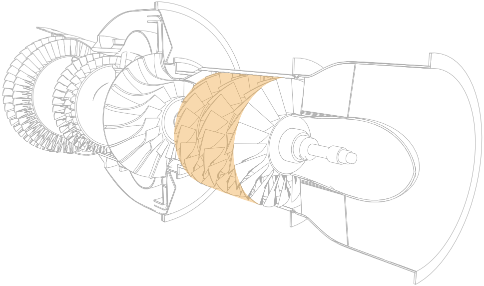
ASME
INTERNATIONAL GAS
TURBINE INSTITUTE

Foreword

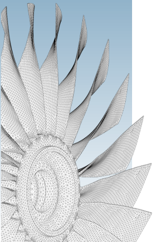


Foreword

- overview of past and on-going research in the field of rotor/stator interactions
- focus on blade-tip/casing interface
  1. why do manufacturers focus on this interface ?
  2. what are the related research challenges ?
  3. what could be the practical outcomes of these works ?
- current bottlenecks and challenges to tackle

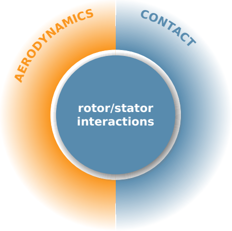


▶ blade-tip/casing interface in the low-pressure compressor of a helicopter engine

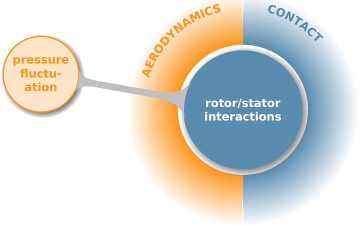


① Rotor/stator interactions

Research context



Research context



- first investigations with respect to unsteady forces due to airfoil passage [1]
- investigations on propagation of wakes through blade rows [2]
- development of experimental and numerical methods in unsteady aerodynamics [3]
- led to experimental works related to pressure fluctuations in turbines [4]

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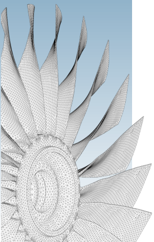
[1] N. H. Kemp et al. *Journal of the Aeronautical Sciences* (1953). DOI: 10.2514/8.2758.

[2] M. D. Lefcort. en. *Journal of Engineering for Power* (1965). DOI: 10.1115/1.3678275.

[3] H. E. Gallus. en. New York, NY: Springer, 1993. DOI: 10.1007/978-1-4613-9341-2_23.

[4] R. P. Dring et al. en. *Journal of Engineering for Power* (1982). DOI: 10.1115/1.3227339.



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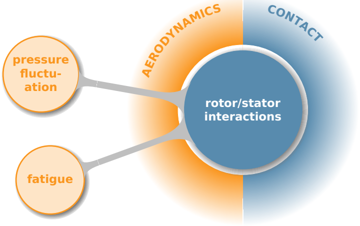

- impact on the fatigue life of bladed components [5, 6]
- numerical/experimental comparisons [7]
- development of numerical methods for mistuned bladed disks [8]

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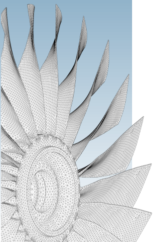
[5] R. Mailach et al. en. *Journal of Turbomachinery* (2004). DOI: 10.1115/1.1791641.

[6] R. Mailach et al. en. *Journal of Turbomachinery* (2004). DOI: 10.1115/1.1791642.

[7] M. B. Schmitz et al. en. Dordrecht: Springer Netherlands, 2006. DOI: 10.1007/1-4020-4605-7_9.

[8] E. Seinturier et al. en. American Society of Mechanical Engineers Digital Collection, 2009. DOI: 10.1115/GT2002-30424.



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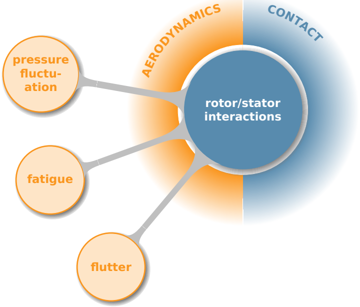

- developments driven by research in a military context[9]
- development of several numerical methods dedicated to flutter calculations: 2D[10] and 3D[11]
- vast amount of research on the subject[12]

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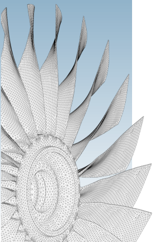
[9] M. F. Platzer et al. 1987, available online.

[10] J. M. Verdon et al. *AIAA Journal* (1982). DOI: 10.2514/3.51186.

[11] L. He et al. en. *Journal of Turbomachinery* (1994). DOI: 10.1115/1.2929436.

[12] K. Isomura et al. en. *Journal of Turbomachinery* (1998). DOI: 10.1115/1.2841746.



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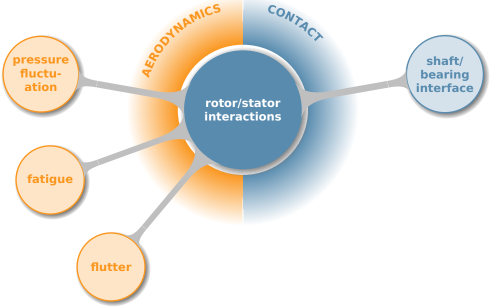

- focus on phenomenological models (Jeffcott rotor)[13]
- evidence of complex dynamics behaviors[14]
- prediction and characterization of whirl/whip motions[15]

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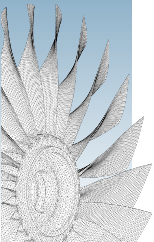
[13] D. W. Childs. en. *Journal of Mechanical Design* (1979). DOI: 10.1115/1.3454114.

[14] F. F. Ehrich. en. *Journal of Vibration, Acoustics, Stress, and Reliability in Design* (1988). DOI: 10.1115/1.3269488.

[15] A. Muszynska. en. *Journal of Sound and Vibration* (1986). DOI: 10.1016/S0022-460X(86)80146-8.



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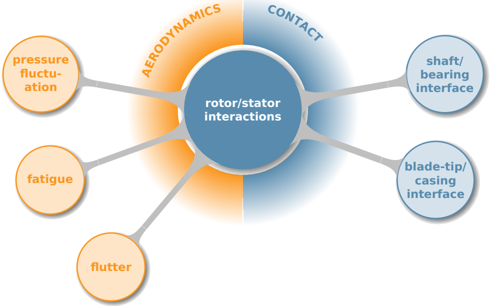

- generalization of shaft/bearing systems to bladed rotors [16]
- extension of the modelling to blades' torsion modes [17]
- development of predictive numerical strategies [18, 19]

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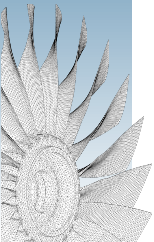
16. J. Padovan et al. en. *Journal of Turbomachinery* (1987). DOI: 10.1115/1.3262143.
17. S. Edwards et al. en. *Journal of Sound and Vibration* (1999). DOI: 10.1006/jsvi.1999.2302.
18. N. Lesaffre et al. en. *European Journal of Mechanics - A/Solids* (2007). DOI: 10.1016/j.euromechsol.2006.11.002.
19. M. Legrand et al. en. *Journal of Sound and Vibration* (2012). DOI: 10.1016/j.jsv.2012.01.017.



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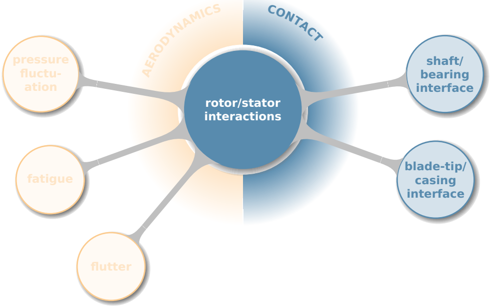

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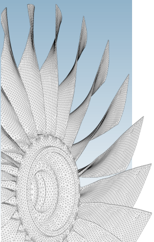
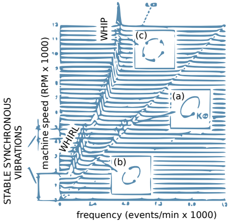


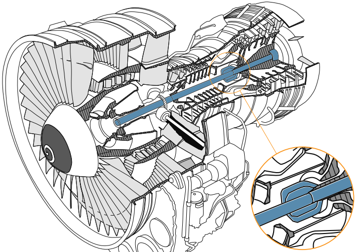
Contact interfaces in aircraft engines

Shaft/bearing

- shaft orbital motions due to mass unbalance or misalignment
- contacts between shaft and supporting bearings
- modelling strategies:
  - single rotors: Black,[20] Jeffcott
  - dual rotors: Childs [21]



- **waterfall diagram** [22]



shaft/bearing

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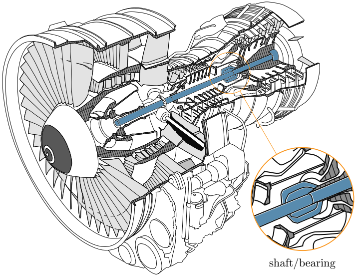
20. H. F. Black. en. *Journal of Mechanical Engineering Science* (2006). DOI: 10.1243/JMES_JOUR_1968_010_003_02.
21. D. W. Childs. en. *Journal of Engineering for Industry* (1976). DOI: 10.1115/1.3439046.
22. A. Muszynska. en. *Journal of Sound and Vibration* (1986). DOI: 10.1016/S0022-460X(86)80146-8.

Contact interfaces in aircraft engines

Shaft/bearing

- shaft orbital motions due to mass unbalance or misalignment
- contacts between shaft and supporting bearings
- modelling strategies:
  - single rotors: Black,[20] Jeffcott
  - dual rotors: Childs [21]
- interface characterized by:
  - non-accidental and accidental configurations
  - impacting components often considered rigid
  - small relative displacements
  - essentially structural dynamics considerations



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20. H. F. Black. en. *Journal of Mechanical Engineering Science* (2006). DOI: 10.1243/JMES_JOUR_1968_010_003_02.
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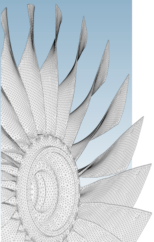
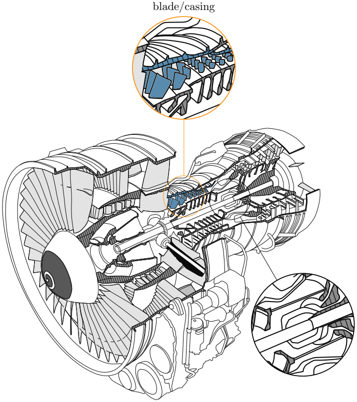
22. A. Muszynska. en. *Journal of Sound and Vibration* (1986). DOI: 10.1016/S0022-460X(86)80146-8.

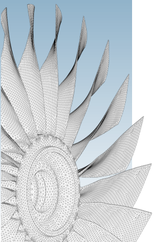


Contact interfaces in aircraft engines

Blade/casing

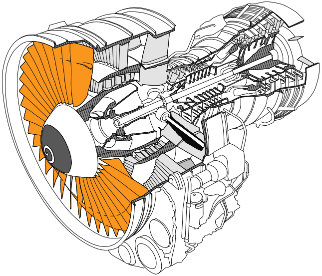
- requires high resolution models for accurate results
- interface characterized by:
  - very high relative speeds (500 m/s)
  - inherently multiphysics: aerodynamic loading + blade vibrations + abradable coating wear + thermomechanics
  - combination of several types of nonlinearities: contact + geometric (fan, turbines)
  - shift of blades′ eigenfrequencies (impact on critical speeds prediction)
- influence of operating clearances on overall efficiency ⇒ **key interface for designers**
- relevant interface at **all stages of an engine**





Fan stage

- specificities
  - large slender blades
  - geometrical nonlinearities may be accounted for
  - use of composite materials
- modal interactions
  - blade and casing exchange energy through repeated structural contacts
  - under specific conditions, geometrical match (same nodal diameter)
- orbital motions in accidental configurations
  - bearing failure
  - forward and backward whirl motions



Interaction phenomena

- modal interaction (dynamics)[23, 24]
- whirl motions (dynamics + inertial effects)[25]

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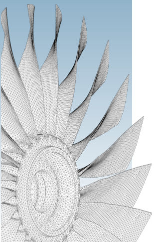
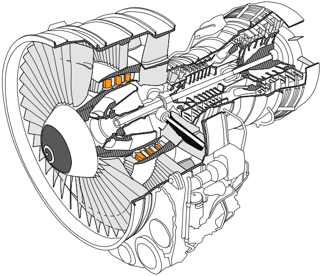
23. P. Schmiechen. en. PhD thesis. 1997, available online.
24. M. Legrand et al. en. *Journal of Sound and Vibration* (2009). DOI: `10.1016/j.jsv.2008.06.019`.
25. N. Salvat et al. en. *International Journal of Non-Linear Mechanics* (2016). DOI: `10.1016/j.ijnonlinmec.2015.10.001`.



Low-pressure compressor

- design considerations
  - operating clearances must be minimized for higher efficiency
  - high cost associated to blade maintenance
- specificities
  - frequent use of abradable coatings (AlSi-Polyester...) [26]
  - rigid casings and usually limited influence of disk modes [27]
- rubbing interactions
  - localized interaction involving a single blade [28]
  - sophisticated wear related physical phenomena (grooving, material transfer...)



Interaction phenomena

- rubbing (blade dynamics + wear)

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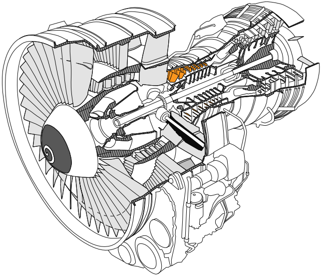
[26] L. T. Shiembob. Technical report. 1975, available online.

[27] A. Millecamps et al. en. American Society of Mechanical Engineers Digital Collection, 2015. DOI: 10.1115/GT2015-42682.

[28] K. E. Turner et al. en. American Society of Mechanical Engineers Digital Collection, 2010. DOI: 10.1115/GT2010-22166.

High-pressure compressor

- design challenges
  - frequent use of blisks
  - disk dynamics must be considered
  - operating clearances must be minimal
- modal interactions
  - requires the modelling of the full bladed disk[29]
  - on-going investigations with respect to inter-stage coupling[30]
- rubbing interactions
  - localized interaction involving a single blade[31]
  - sophisticated wear related physical phenomena (grooving, material transfer...)



Interaction phenomena

- modal interaction (dynamics + wear)
- rubbing (blade dynamics + wear + thermomechanics)

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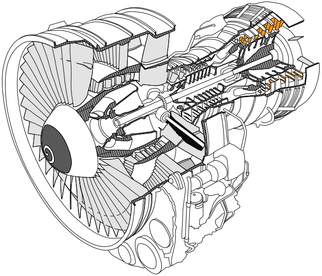
[29] M. Legrand et al. en. *Journal of Sound and Vibration* (2012). DOI: 10.1016/j.jsv.2012.01.017.

[30] G. Battiato et al. en. *Journal of Engineering for Gas Turbines and Power* (2018). DOI: 10.1115/1.4038348.

[31] A. Batailly et al. en. *Journal of Sound and Vibration* (2016). DOI: 10.1016/j.jsv.2016.03.016.

Turbine stages

- design challenges
  - use of new heat resistant CMC materials (stator shroud segments...)[32]
  - more complex blade geometries
  - use of thermally sprayed coatings on blades[33]
- specificities
  - extreme temperatures
  - non-negligible blade to blade contacts (shrouds...)
- rubbing interactions
  - localized interaction involving a single blade
  - sophisticated wear related physical phenomena (grooving, material transfer...)[34]



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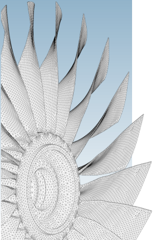
[32] F. Nyssen et al. en. *Journal of Sound and Vibration* (2020). DOI: 10.1016/j.jsv.2019.115040.

[33] S. Colón et al. en. American Society of Mechanical Engineers Digital Collection, 2019. DOI: 10.1115/GT2019-90886.

[34] R. K. Schmid. en. PhD thesis. ETH Zurich, 1997. DOI: 10.3929/ethz-a-001809249.



Research context

Review articles in the field of rotor/stator contacts

- 1969 - shaft/bearing interface [35]
- 1989 - shaft/bearing interface [36]
- 2010 - shaft/bearing interface [37]
- 2013 - blade/casing interface (mostly) [38]
- 2016 - blade/casing interface [39]
- 2020 - both interfaces [40]

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[35] A. D. Dimarogonas et al. en. *Wear* (1969). DOI: 10.1016/0043-1648(69)90037-4.

[36] A. Muszynska. *Rotor-to-stationary element sub-related vibration phenomena in rotating machinery: literature survey* (1989).

[37] S. Ahmad. *Journal of Vibration and Control* (2010). DOI: 10.1177/1077546309341605.

[38] G. Jacquet-Richardet et al. en. *Mechanical Systems and Signal Processing* (2013). DOI: 10.1016/j.ymssp.2013.05.010.

[39] H. Ma et al. en. *Nonlinear Dynamics* (2016). DOI: 10.1007/s11071-015-2535-x.

[40] K. Prabith et al. en. *Nonlinear Dynamics* (2020). DOI: 10.1007/s11071-020-05832-y.

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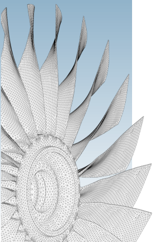
Purpose of this presentation

- emphasize the specificity of the blade/casing interface
- contextualize research works with respect to industrial needs
- bridge research on wear modelling and research on abradable coatings with recent numerical developments



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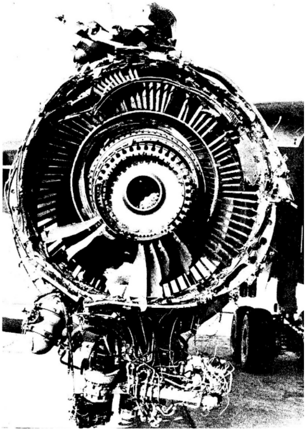
40. K. Prabith et al. en. *Nonlinear Dynamics* (2020). DOI: 10.1007/s11071-020-05832-y.

Understanding and predicting blade/casing contacts

Multifaceted challenges

- aerospace industry: major safety concerns [41], [42], [43]
- power generation: maintenance and cost issues [44]
- automative industry: mostly a performance issue

Most of the research works detailed in the remainder are related to the aerospace industry.



▶ photo from [41]

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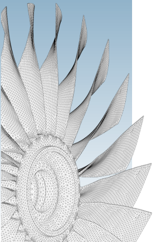
[41] H. R. John et al. Technical report. National Transport Safety Board, 1975, available online.

[42] C. A. Christie. Aviation Rulemaking Advisory Committee. FAA, 1996, available online.

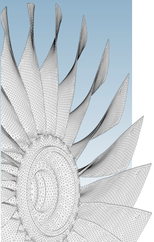
[43] Technical report. Australian Transport Safety Bureau, 2008, available online.

[44] C. J. Hulme et al. en. American Society of Mechanical Engineers Digital Collection, 2015. DOI: 10.1115/GT2015-43312.



Understanding and predicting blade/casing contacts

full scale exp. setups



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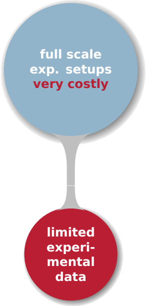
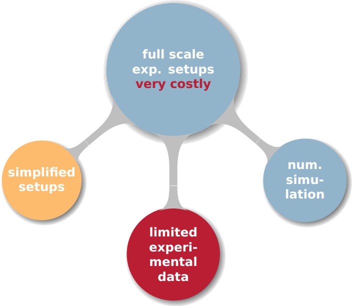


full scale exp. setups very costly

limited experimental data

Understanding and predicting blade/casing contacts



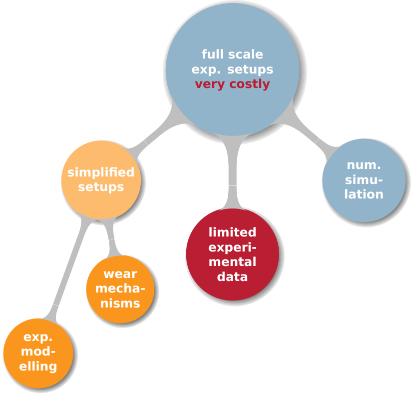
full scale exp. setups **very costly**

simplified setups

limited experimental data

num. simulation

Understanding and predicting blade/casing contacts



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simplified setups

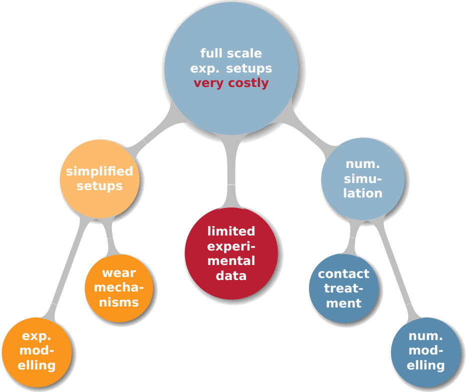
limited experimental data

num. simulation

wear mechanisms

exp. modelling

Understanding and predicting blade/casing contacts



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simplified setups

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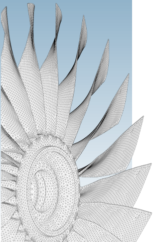
num. simulation

contact treatment

num. modelling



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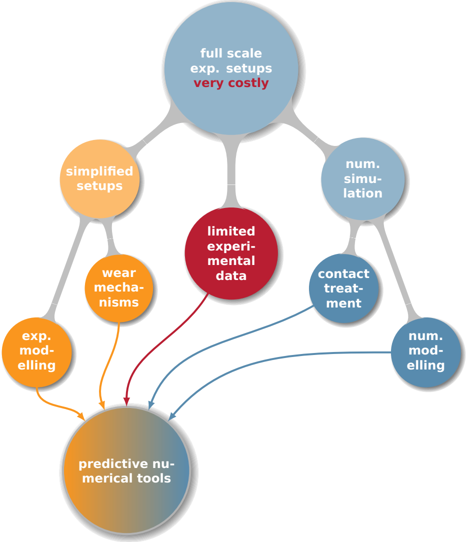

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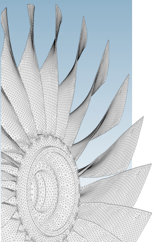
wear mechanisms

exp. modelling

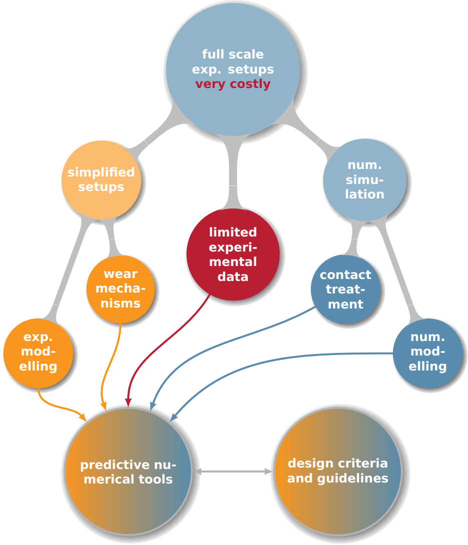
contact treatment

num. modelling

predictive numerical tools



Understanding and predicting blade/casing contacts



full scale exp. setups very costly

simplified setups

limited experimental data

num. simulation

wear mechanisms

exp. modelling

contact treatment

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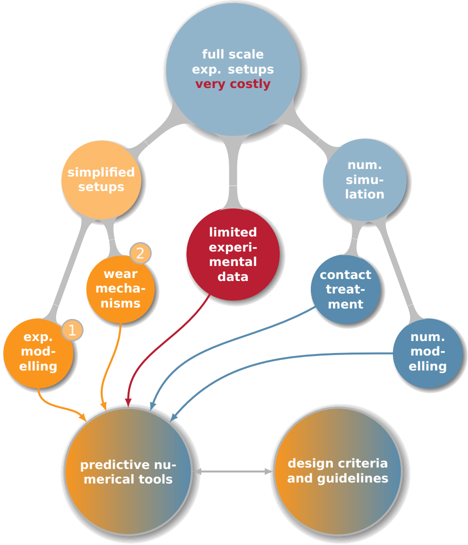
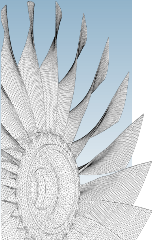
predictive numerical tools

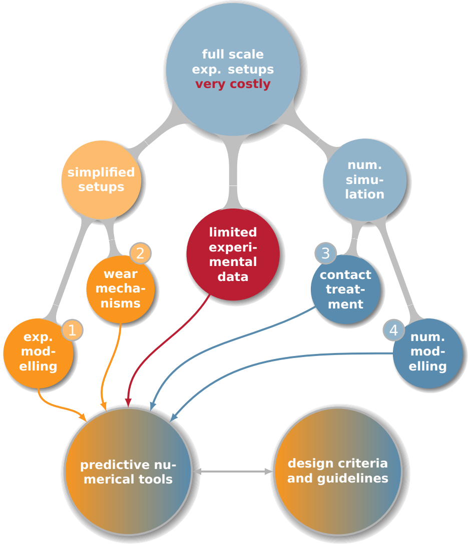
design criteria and guidelines

Understanding and predicting blade/casing contacts





Understanding and predicting blade/casing contacts



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1 exp. modelling

2 wear mechanisms

3 contact treatment

4 num. modelling

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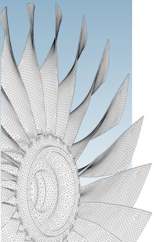
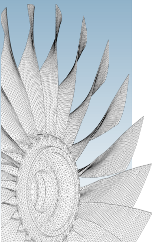
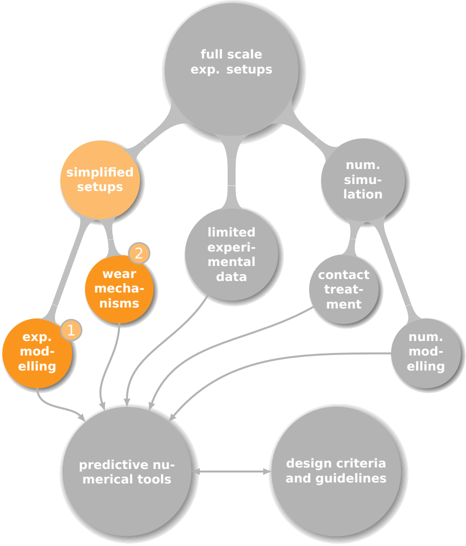
design criteria and guidelines



② Research investigations





full scale exp. setups

simplified setups

limited experimental data

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exp. modelling

1

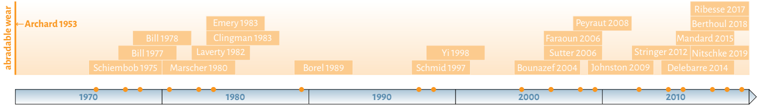
contact treatment

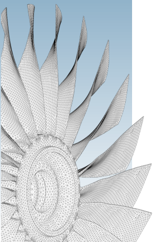
num. modelling

predictive numerical tools

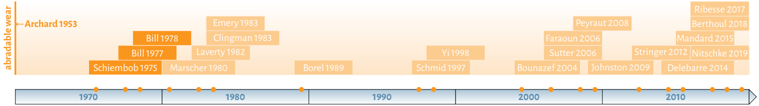
design criteria and guidelines

Abradable coating mechanical properties



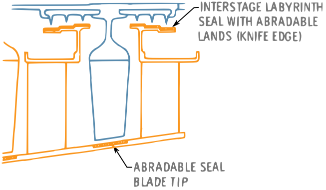


Abradable coating mechanical properties



Development of abradable materials

- knife edge seals applications[45]
- various alloys and honeycomb structures
- compromise to be found between:
  - abradability of the material[46]
  - erosion performance and thermal resistance



[47]

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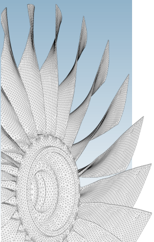
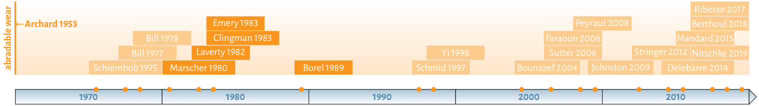
45. L. T. Shiembob. Technical report. 1975, available online.

46. R. C. Bill et al. en. 1978, available online.

47. R. C. Bill et al. en. *Journal of Lubrication Technology* (1977). DOI: 10.1115/1.3453236.

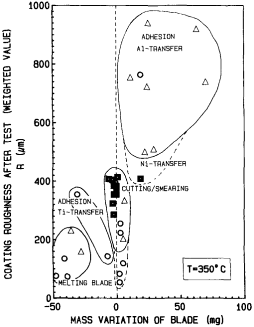


Abradable coating mechanical properties



Wear mechanisms in gas turbines

- growing interest for blade/casing contacts [48] and ceramic seals [49]
- first description of wear models [50] and detailed description of thermomechanical phenomena
- description of distinct wear mechanisms in gas turbines abradable seals [51]



▶ wear mechanisms identified in [51]

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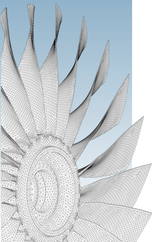
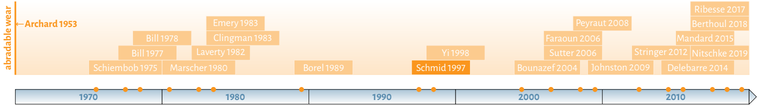
48. A. F. Emery et al. en. *Wear* (1983). DOI: 10.1016/0043-1648(83)90248-X.
49. D. L. S. Clingman. *Lubrication Engineering* (1983, available online).
50. W. D. Marscher. en. *Wear* (1980). DOI: 10.1016/0043-1648(80)90278-1.
51. M. O. Borel et al. en. *Surface and Coatings Technology* (1989). DOI: 10.1016/0257-8972(89)90046-7.



Abradable coating mechanical properties



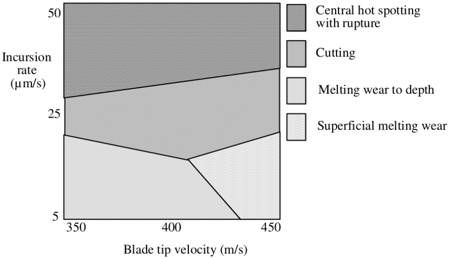
abradable wear

←Archard 1953

Schiembob 1975 · Bill 1977 · Bill 1978 · Marscher 1980 · Laverty 1982 · Emery 1983 · Clingman 1983 · Borel 1989 · Schmid 1997 · Yi 1998 · Bounazef 2004 · Sutter 2006 · Faraoun 2006 · Peyraut 2008 · Johnston 2009 · Stringer 2012 · Delebarre 2014 · Mandard 2015 · Ribesse 2017 · Berthoul 2018 · Nitschke 2019

Performance analysis of abradable seals [52]

- focus on purely material aspects (no vibration aspects)
- expansion and generalization of Borel's work
- use of wear maps for performance comparisons



▶ wear map of AlSi abradable coatings at 400°C from [52]

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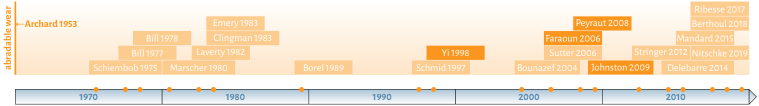
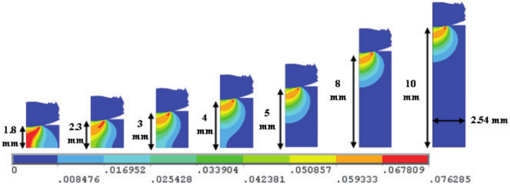
[52] R. K. Schmid. en. PhD thesis. ETH Zurich, 1997. DOI: 10.3929/ethz-a-001809249.

Abradable coating mechanical properties



Static / low-speed characterization of abradable seals

- use of scratch tests [53] and HR15Y [54]
- first numerical models (static, FE) of abradable seals [55]
- analysis of distinct types of wear (adhesive, abrasive, oxidation...)



▶ results of finite element computations carried out in [55]

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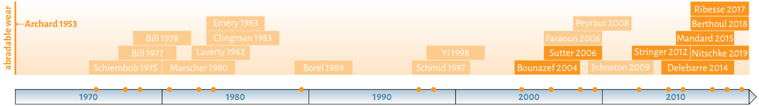
[54] M. Yi et al. (1998, available online).

[55] F. Peyraut et al. en. *Int. J. for Simulation and Multidisciplinary Design Optimization* (2008). DOI: 10.1051/ijsmdo:2008028.

[56] H. I. Faraoun et al. en. *Surface and Coatings Technology* (2006). DOI: 10.1016/j.surfcoat.2005.11.105.

Abradable coating mechanical properties



←Archard 1953
Schiembob 1975
Bill 1977
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Marscher 1980
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Clingman 1983
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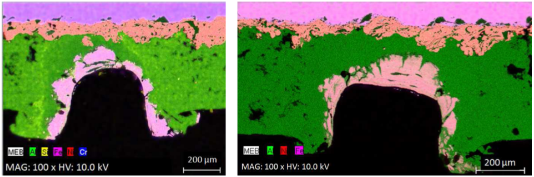
Schmid 1997
Yi 1998
Bounazef 2004
Sutter 2006
Faraoun 2006
Peyraut 2008
Johnston 2009
Stringer 2012
Delebarre 2014
Mandard 2015
Ribesse 2017
Berthoul 2018
Nitschke 2019

Dynamic characterization, interaction with blade dynamics

- rubbing is now accepted as a non-accidental event by manufacturers [57]
- growing interest for the vibration behaviour of the impacting component [58]
- investigations on complex transfer phenomena [59]
- more realistic experimental setups aiming at engine-like conditions [60]



▶ Energy-dispersive X-ray spectroscopy of an abradable coating shown in [60]

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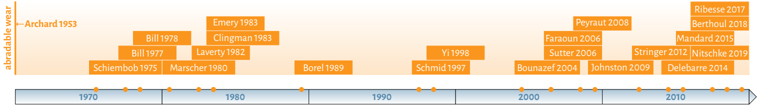
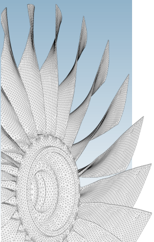
58. B. Berthoul et al. en. *Mechanical Systems and Signal Processing* (2018). DOI: 10.1016/j.ymssp.2017.05.020.
59. R. Mandard et al. en. *Tribology International* (2015). DOI: 10.1016/j.triboint.2014.01.026.
60. C. Delebarre et al. en. *Wear* (2014). DOI: 10.1016/j.wear.2014.04.023.
61. S. Nitschke et al. en. *Wear* (2019). DOI: 10.1016/j.wear.2018.12.072.

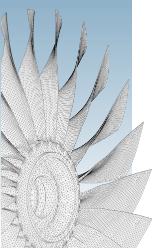
Abradable coating mechanical properties



Over time: two types of experimental setups

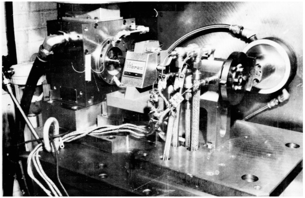
- investigating wear mechanisms (1975-today)
  - incursion rate
  - relative interaction speed
  - material type
- focusing on blade vibrations and complex chemical interactions (2000-today)
  - variation of blade profile
  - engine-like conditions



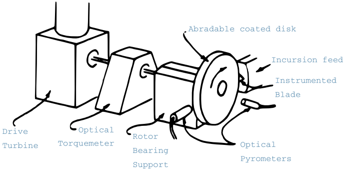


Simplified experimental setups

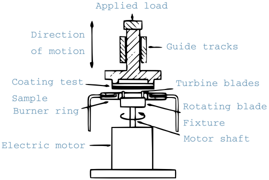
- 1975: NASA / Pratt & Whitney[62]
  - development of abradable coatings for high temperature environments (850 - 1 100°C)
  - compromise between erosion performance and abradability



- 1982: Pratt & Whitney[63]
  - interaction with fibermetal strips
  - single blade, room temperature



- 1982: Metco[64]
  - setup for ceramic abradable turbine seal
  - constant contact over time
  - heated coating (1 100°C)



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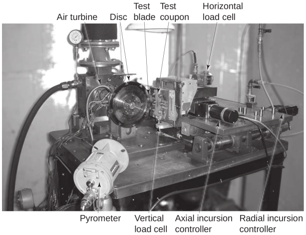
[62] L. T. Shiembob. Technical report. 1975, available online.

[63] W. F. Laverty. en. *Wear* (1982). DOI: 10.1016/0043-1648(82)90137-5.

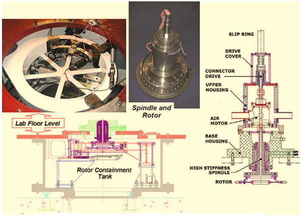
[64] E. Novinski et al. en. *Thin Solid Films* (1982). DOI: 10.1016/0040-6090(82)90018-9.

Simplified experimental setups

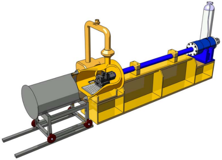
- 2008: Gas Turbine Laboratory (CNRC)[65]
  - effect of temperature on abradable seals
  - influence of other parameters including:
    - blade rotation speed
    - incursion rate



- 2002 / 2010: Gas Turbine Laboratory (OSU - GE)[66]
  - use of actual aircraft engine blades
  - engine-like conditions (speed, contact profile)
  - Rotor-Blade Rub database



- 2012: ENIM - Snecma[67]
  - titanium projected on an abradable layer
  - focus on the measurement of cutting forces
  - large incursion rates



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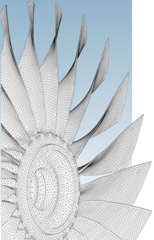
[65] A. Dadouche et al. en. American Society of Mechanical Engineers Digital Collection, 2009. DOI: 10.1115/GT2008-51228.

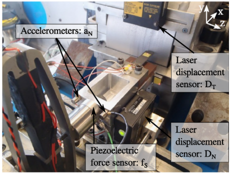
[66] C. Padova et al. en. *Journal of Turbomachinery* (2011). DOI: 10.1115/1.4000539.

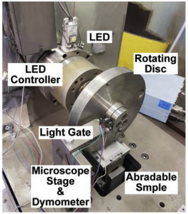
[67] M. Cuny et al. en. *Experimental Mechanics* (2014). DOI: 10.1007/s11340-013-9780-z.

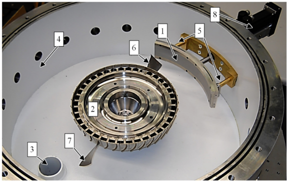


Simplified experimental setups

- 2015: ONERA - Snecma[68]
  - rectangular plate / coated cylinder
  - focus on the blade vibrations
- 2018: University of Sheffield[69]
  - small simplified blades
  - variation of wear processes with time
- 2019: Technische Universität Dresden - Rolls-Royce[70]
  - use of actual aircraft engine blades
  - multiple impacts per revolution are possible
  - engine-like boundary conditions







1: 72° casing segment with abradable liner material
2: bladed disk mounted on spindle drive
3: vacuum access
4: steel containment
5: linear guidance
6: test blade
7: counter blade
8: linear feed motor for axial movement of the casing segment

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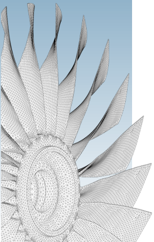
68. R. Mandard et al. en. *Tribology International* (2015). DOI: 10.1016/j.triboint.2014.01.026.
69. B. Zhang et al. en. *Wear* (2019). DOI: 10.1016/j.wear.2019.01.034.
70. S. Nitschke et al. en. *Wear* (2019). DOI: 10.1016/j.wear.2018.12.072.



Simplified experimental setups

- 1978: Lewis Research Center (NASA)[71]
- 2002: Central South University of Technology[72]
- 2004: UTBM[73]
- 2007: Alstom[74]
- 2014: ENIT - Snecma[75]
  - focus on labyrinth seals
  - investigations on material transfer

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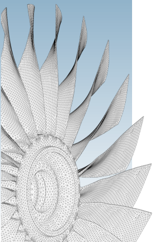
71. R. C. Bill et al. en. 1978, available online.
72. Y. Maozhong et al. en. *Wear* (2002). DOI: 10.1016/S0043-1648(01)00681-0.
73. M. Bounazef et al. en. *Materials Letters* (2004). DOI: 10.1016/j.matlet.2004.02.049.
74. U. Rathmann et al. en. American Society of Mechanical Engineers Digital Collection, 2009. DOI: 10.1115/GT2007-27724.
75. C. Delebarre et al. en. *Wear* (2014). DOI: 10.1016/j.wear.2014.04.023.



Summary of experimental investigations

- 1975-2020: from coating investigations to the characterization of lighter materials (CMC turbine shrouds [76])
    - initial focus on abradable coating mechanical properties in a context where contacts were accidental
    - growing interest for the influence of contacts on the blade response
    - integration of contacts within design certification process

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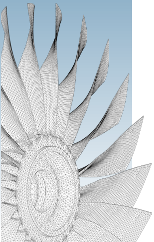
[76] F. Nyssen et al. en. *Journal of Sound and Vibration* (2020). DOI: 10.1016/j.jsv.2019.115040.



Summary of experimental investigations

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  - initial focus on abradable coating mechanical properties in a context where contacts were accidental
  - growing interest for the influence of contacts on the blade response
  - integration of contacts within design certification process
- abradable coatings can now be used in every stage of a turbomachine
- which yields higher efficiency (gas turbines and aircraft engines)
- variety of technological solutions available (AlSi-Polyester, ceramic, honeycomb)...
- ...that requires stage by stage analyses of rubbing interactions...
- and motivates the development of realistic setups featuring actual blades and interchangeable abradable samples

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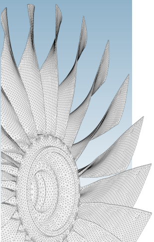
[76] F. Nyssen et al. en. *Journal of Sound and Vibration* (2020). DOI: 10.1016/j.jsv.2019.115040.



Summary of experimental investigations

Challenges and open research questions

- many wear mechanisms are still not fully understood
    - material transfer (blade on casing or casing on blade)
    - grooving phenomena in the abradable seal
- specific challenges in turbine stages where blades feature protective coatings[77]
- mechanical characterization of abradable coatings (modelling) is an open question[78]

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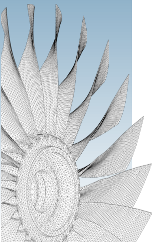
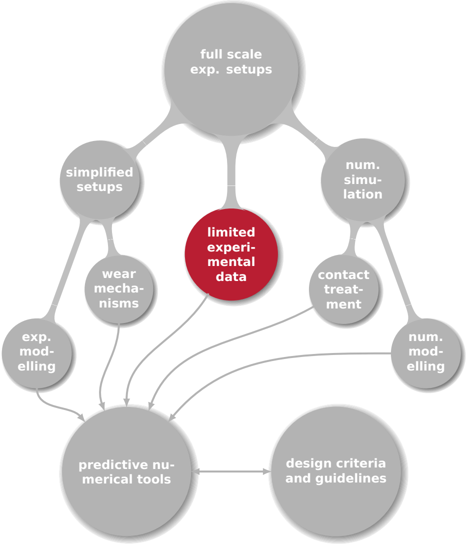
[77] S. Colón et al. en. American Society of Mechanical Engineers Digital Collection, 2019. DOI: 10.1115/GT2019-90886.

[78] S. Skiba et al. en. *Journal of Dynamic Behavior of Materials* (2020). DOI: 10.1007/s40870-020-00242-y.





full scale exp. setups

simplified setups

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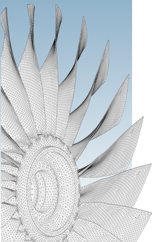

Full-scale experimental setups

- no existing facility for multi-stage full-scale experimental setups
- existing experimental setups feature a single stage
- most investigations are carried out under vacuum
  - rubbing interaction within low-pressure compressor [79]
  - rubbing interaction within high-pressure compressor [80]
  - Large Spin Pit Facility (LSPF) at OSU [81]
  - PHARE experimental setup in ECL [82]

picture of the Equipex-PHARE test bench (source)

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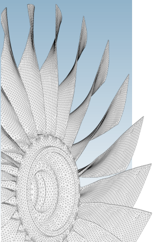
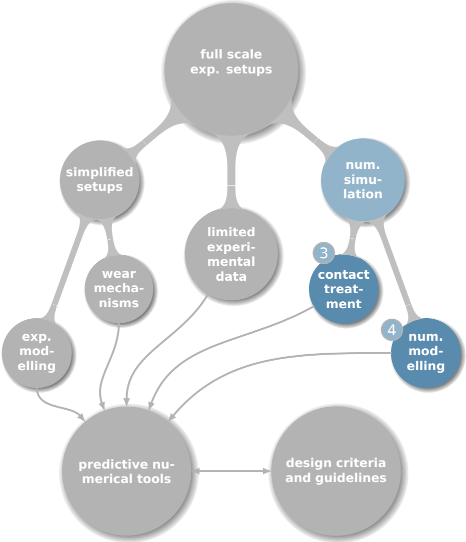
[79] A. Millecamps et al. en. American Society of Mechanical Engineers Digital Collection, 2010. DOI: 10.1115/DETC2009-86842.

[80] A. Batailly et al. en. *Journal of Sound and Vibration* (2016). DOI: 10.1016/j.jsv.2016.03.016.

[81] N. Langenbrunner et al. en. *Journal of Engineering for Gas Turbines and Power* (2015). DOI: 10.1115/1.4028685.

[82] F. Thouverez. fr. *PHARE Plateforme macHines tournantes pour la mAîtrise des Risques Environnementaux* (). DOI: https://www.ec-lyon.fr/campus/vie-campus/grands-projets/equipex-phare.





full scale exp. setups

simplified setups

limited experimental data

num. simulation

wear mechanisms

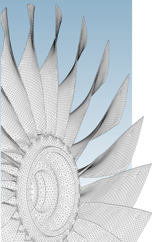
exp. modelling

3 contact treatment

4 num. modelling

predictive numerical tools

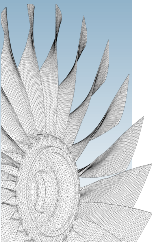
design criteria and guidelines



Numerical framework

Each type of blade/casing interaction requires a specific numerical framework:

rotor	stator	interactions	required features
1 blade	rigid (flexible)	rubbing	contact treatment structural dynamics wear
full bladed disk	flexible	modal interaction	contact treatment structural dynamics two flexible components
bladed disk + shaft	flexible (rigid)	whirl motions	contact treatment structural dynamics inertial effects



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Modal interaction

- full bladed disk required
- both stator and rotor must be flexible[83]
- significant computational cost ⇒ use of phenomenological models[84]

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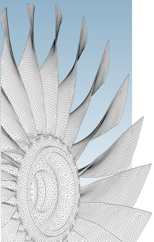
[83] P. Schmiechen. en. PhD thesis. 1997, available online.

[84] M. Legrand et al. en. *Journal of Sound and Vibration* (2009). DOI: 10.1016/j.jsv.2008.06.019.



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bladed disk + shaft	flexible (rigid)	whirl motions	contact treatment structural dynamics inertial effects

Rubbing

- very precise modelling of the blade/casing interface is required[85]
- live update of clearances due to wear is key[86, 87]
- industrial 3D finite element models are usually considered[88]

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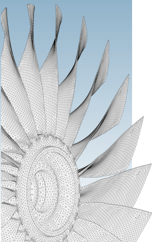
[85] M. Legrand et al. en. *Journal of Sound and Vibration* (2012). DOI: 10.1016/j.jsv.2012.01.017.

[86] M. Legrand et al. en. American Society of Mechanical Engineers Digital Collection, 2010. DOI: 10.1115/DETC2009-87669.

[87] R. J. Williams. en. American Society of Mechanical Engineers Digital Collection, 2012. DOI: 10.1115/GT2011-45495.

[88] A. Batailly et al. en. *Journal of Engineering for Gas Turbines and Power* (2012). DOI: 10.1115/1.4006446.



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bladed disk + shaft	**flexible (rigid)**	**whirl motions**	**contact treatment** **structural dynamics** **inertial effects**

Whirl motions

- inertial effects cannot be neglected due to orbital motions
- both phenomenological [89] and full FE [90] models are considered
- published data for such interactions are very scarce

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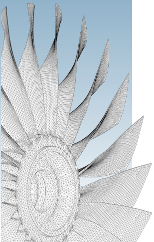
[89] M.-O. Parent et al. en. American Society of Mechanical Engineers Digital Collection, 2014. DOI: 10.1115/GT2014-25253.

[90] N. Salvat et al. en. *International Journal of Non-Linear Mechanics* (2016). DOI: 10.1016/j.ijnonlinmec.2015.10.001.



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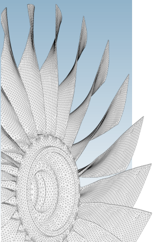

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Numerical and theoretical challenges

1. solution strategy
   - periodic (synchronous) motions can reasonably be expected in aircraft engines...
   - ...but transient phenomena seem key for the rise of critical interactions [91]
   - two types of strategies: frequency methods [92, 93] vs. time integration [94]
2. contact mechanics
3. modelling

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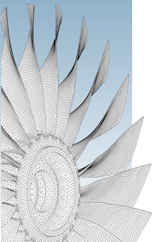
[91] A. Millecamps et al. en. American Society of Mechanical Engineers Digital Collection, 2010. DOI: 10.1115/DETC2009-86842.

[92] G. von Groll et al. en. *Journal of Sound and Vibration* (2001). DOI: 10.1006/jsvi.2000.3298.

[93] L. Salles et al. en. American Society of Mechanical Engineers Digital Collection, 2010. DOI: 10.1115/GT2010-23264.

[94] N. J. Carpenter et al. en. *International Journal for Numerical Methods in Engineering* (1991). DOI: 10.1002/nme.1620320107.



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Numerical and theoretical challenges

1. solution strategy
2. contact mechanics
    - inherently nonlinear mechanical system ⇒ no unified theoretical framework[95]
    - numerical management of non-smooth contact algorithms yields distinct *ad hoc* strategies[96, 97]
    - severe bottleneck for industry-ready analyses[98]

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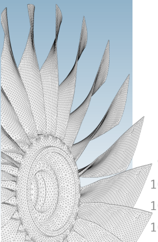
[95] S. K. Sinha. en. *International Journal of Non-Linear Mechanics* (2005). DOI: `10.1016/j.ijnonlinmec.2004.05.019`.

[96] C. Yoong et al. en. *Nonlinear Dynamics* (2018). DOI: `10.1007/s11071-017-4025-9`.

[97] N. Lesaffre et al. en. *European Journal of Mechanics - A/Solids* (2007). DOI: `10.1016/j.euromechsol.2006.11.002`.

[98] A. Batailly et al. en. *Journal of Engineering for Gas Turbines and Power* (2015). DOI: `10.1115/1.4028263`.



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bladed disk + shaft	flexible (rigid)	whirl motions	contact treatment structural dynamics inertial effects

Numerical and theoretical challenges

1. solution strategy
2. contact mechanics
3. modelling
    - high definition finite element models are required (several millions of dof)[99]
    - models must include one (or more) nonlinear interfaces[100]
    - possible geometric nonlinearities which yield additional computational cost[101]
    - sensitivity of the results design parameters[102](mistuning...)

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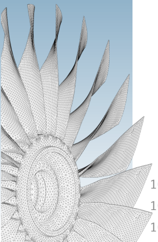
[99] E. P. Petrov. en. American Society of Mechanical Engineers Digital Collection, 2013. DOI: 10.1115/GT2012-68474.

[100] E. P. Petrov. en. American Society of Mechanical Engineers Digital Collection, 2008. DOI: 10.1115/GT2004-53894.

[101] M. Balmaseda et al. en. American Society of Mechanical Engineers Digital Collection, 2019. DOI: 10.1115/GT2019-90813.

[102] A. M. Panunzio et al. en. American Society of Mechanical Engineers Digital Collection, 2015. DOI: 10.1115/GT2015-43560.



Solution strategy

Context

- the design of mechanical systems with joint interfaces, friction or structural contacts is today a major challenge for engineers and researchers
- lack of theoretical framework (no equivalent of modal analysis in a nonlinear context)
- design relevant quantities (eigenfrequencies, mode shapes) are dependent on the amount of energy within the system
- vast amount of theoretical research work in this area:
  - nonlinear normal modes [103, 104]
  - nonsmooth modal analysis [105]

Specificity of the blade/casing interface

- synchronous excitation
- very small incursion rates
- inherently multi-physical
- very high relative speeds

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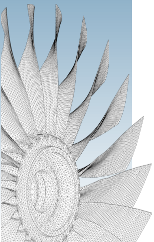
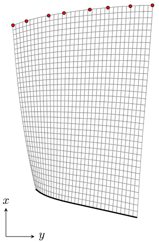
103. G. Kerschen et al. en. *Mechanical Systems and Signal Processing* (2009). DOI: 10.1016/j.ymssp.2008.04.002.
104. M. Krack. en. *Computers & Structures* (2015). DOI: 10.1016/j.compstruc.2015.03.008.
105. A. Thorin et al. *SIAM Journal on Applied Dynamical Systems* (2017). DOI: 10.1137/16M1081506.



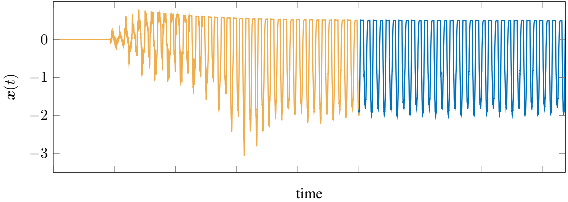
Solution strategy

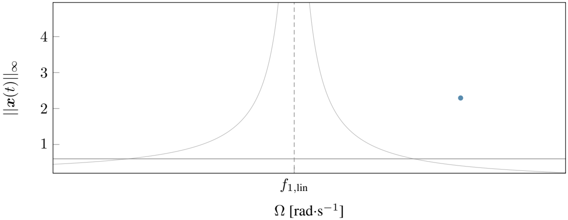
Two paradigms

- time integration based methods
- frequency methods



one angular speed:





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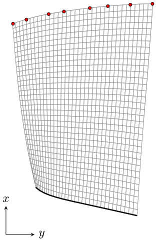
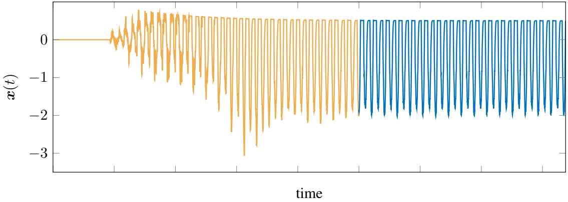
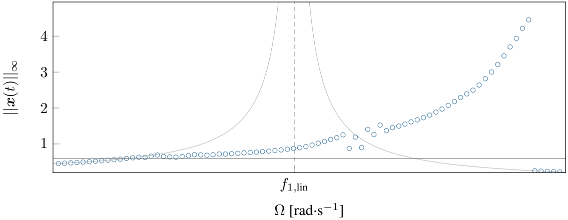


one angular speed:



several angular speeds (sequential continuation $\Omega \nearrow$):



Solution strategy

Two paradigms

- time integration based methods
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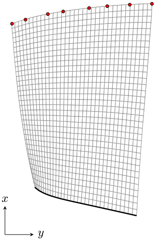
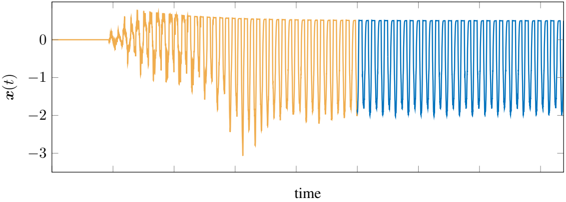


one angular speed:



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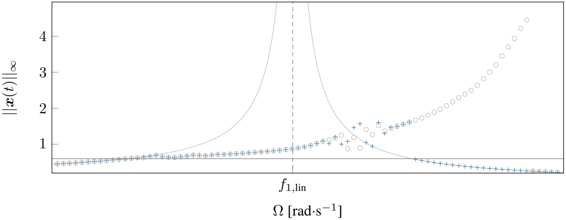

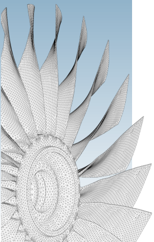

Solution strategy

Two paradigms

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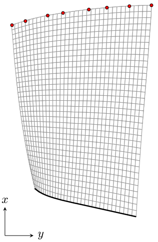
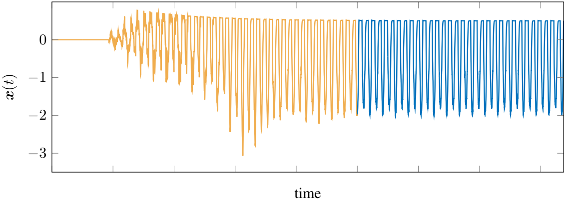
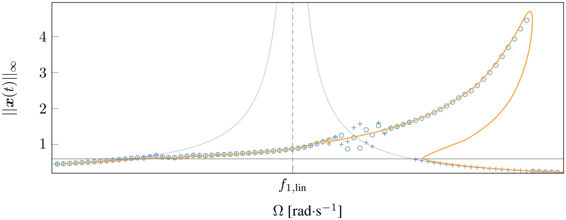
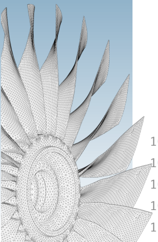


one angular speed:



harmonic balance method (arc-length continuation):

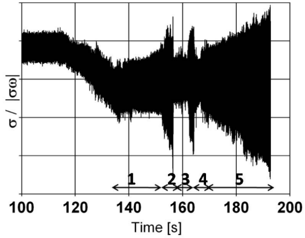




Solution strategy

Time integration methods

- experimental observations of transient phenomena and diverging interactions [106] have motivated the use of time integration
- both implicit [107] and explicit [108] time marching algorithms have been employed…
- …with a variety of contact algorithms [109, 110]



- stress signal within a blade during an interaction (Ω constant for $t > 140$ s) [106]

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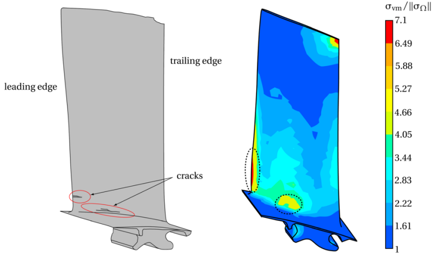
106. A. Millecamps et al. en. American Society of Mechanical Engineers Digital Collection, 2010. DOI: 10.1115/DETC2009-86842.
107. L. Papeleux. 2020, METAFOR website.
108. A. Millecamps et al. en. American Society of Mechanical Engineers Digital Collection, 2015. DOI: 10.1115/GT2015-42682.
109. N. J. Carpenter et al. en. *International Journal for Numerical Methods in Engineering* (1991). DOI: 10.1002/nme.1620320107.
110. A. Thorin et al. en. *Journal of Engineering for Gas Turbines and Power* (2019). DOI: 10.1115/1.4040857.

Solution strategy

Time integration methods

- multiple confrontations of numerical predictions with experimental results:
  - low-pressure compressor [111, 112]
  - high-pressure compressor [113]
  - turbine stages [114]
- results provide a quantitative view of the problem



- experimental and numerical results shown in [111]: stresses in rotor

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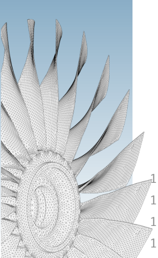
[111] A. Batailly et al. en. *Journal of Engineering for Gas Turbines and Power* (2012). DOI: 10.1115/1.4006446.

[112] Q. Agrapart et al. en. *Journal of Sound and Vibration* (2019). DOI: 10.1016/j.jsv.2019.114869.

[113] A. Batailly et al. en. *Journal of Sound and Vibration* (2016). DOI: 10.1016/j.jsv.2016.03.016.

[114] F. Nyssen et al. en. *Journal of Sound and Vibration* (2020). DOI: 10.1016/j.jsv.2019.115040.



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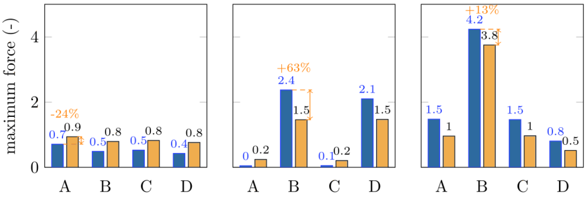

- experimental and numerical results shown in [114]: efforts levels in stator

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[111] A. Batailly et al. en. *Journal of Engineering for Gas Turbines and Power* (2012). DOI: 10.1115/1.4006446.

[112] Q. Agrapart et al. en. *Journal of Sound and Vibration* (2019). DOI: 10.1016/j.jsv.2019.114869.

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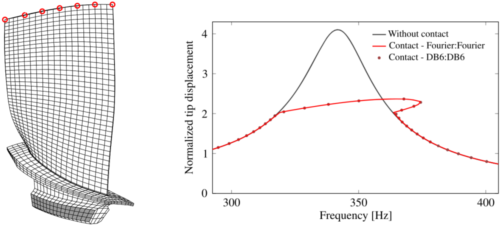
[114] F. Nyssen et al. en. *Journal of Sound and Vibration* (2020). DOI: 10.1016/j.jsv.2019.115040.

Solution strategy

Frequency methods

- usual numerical methodologies for shaft/bearing contacts [115]
- structural contacts between rotor and stator yield a synchronous excitation [116]
- many research works dedicated to the Harmonic Balance Method (HBM): multiharmonic analysis,[117] quasi-periodic HBM,[118] wavelet-based HBM [119]...
- special interest for HBM in turbomachinery (HBM Tutorial ASME Turbo Expo 2018 [120])



▶ **model and results from [119]: finite element mesh and frequency response function of the system**

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[115] G. von Groll et al. en. *Journal of Sound and Vibration* (2001). DOI: 10.1006/jsvi.2000.3298.

[116] H. Ma et al. en. *Journal of Sound and Vibration* (2015). DOI: 10.1016/j.jsv.2014.10.020.

[117] E. P. Petrov. en. American Society of Mechanical Engineers Digital Collection, 2013. DOI: 10.1115/GT2012-68474.

[118] L. Peletan et al. en. *Nonlinear Dynamics* (2014). DOI: 10.1007/s11071-014-1606-8.

[119] S. Jones et al. en. *International Journal for Numerical Methods in Engineering* (2015). DOI: 10.1002/nme.4807.

[120] M. Krack et al. en. American Society of Mechanical Engineers Digital Collection, 2018.

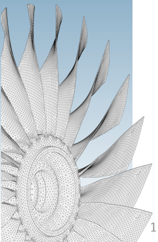
Solution strategy

Time integration

- + easy to implement and wide compatibility with contact algorithms
- + easy extension to multiphysics
- + any type of response may be computed (diverging, chaotic[121]...) as well as transient
- + validation by confrontation to experimental results
- – results dependent on initial conditions
- – a qualitative understanding of the system has a very high computational cost

Frequency methods

- + provides a qualitative understanding of the system
- + easily allows to assess the stability of a solution
- + prediction of hazardous bifurcations and multiple stable branches of solutions
- + computationally efficient but...
- – ...fairly difficult to implement and CPU times explode with the number of nonlinear dof
- – Fourier based analyses are sensitive to the Gibbs phenomenon



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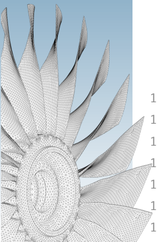
[121] A. Batailly et al. en. American Society of Mechanical Engineers Digital Collection, 2014. DOI: 10.1115/GT2014-25675.



Solution strategy: research directions

Time integration

- variable-order fractional operators offer promising avenues for faster computations [122]
- new contact treatment algorithms to prevent numerical artifacts [123]
- accounting for stochastic components: mistuning [124]

Frequency methods

- development of new algorithms to better handle contact nonlinearities (nonsmooth)
- path-following [125] and continuation [126] techniques
- accounting for stochastic components: uncertain nonlinear normal modes [127] and mistuning [128]

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[122] **S. Patnaik et al.** en. American Society of Mechanical Engineers Digital Collection, 2019. DOI: 10.1115/DETC2019-97944.

[123] **A. Thorin et al.** en. *Journal of Engineering for Gas Turbines and Power* (2019). DOI: 10.1115/1.4040857.

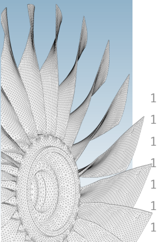
[124] **J. Joachim et al.** en. *Journal of Engineering for Gas Turbines and Power* (2020). DOI: 10.1115/1.4047780.

[125] **A. Simon Chong Escobar et al.** en. *Journal of Computational and Nonlinear Dynamics* (2017). DOI: 10.1115/1.4036114.

[126] **L. Salles et al.** en. *Nonlinear Dynamics* (2016). DOI: 10.1007/s11071-016-3003-y.

[127] **A. M. Panunzio et al.** en. American Society of Mechanical Engineers Digital Collection, 2015. DOI: 10.1115/GT2015-43560.

[128] **C. Joannin et al.** en. *Journal of Engineering for Gas Turbines and Power* (2016). DOI: 10.1115/1.4031886.



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Both strategies are required: time integration does not provide the required qualitative understanding of the system and frequency methods are currently not compatible with large numerical models and do not capture transient phenomena.

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[122] S. Patnaik et al. en. American Society of Mechanical Engineers Digital Collection, 2019. DOI: 10.1115/DETC2019-97944.

[123] A. Thorin et al. en. *Journal of Engineering for Gas Turbines and Power* (2019). DOI: 10.1115/1.4040857.

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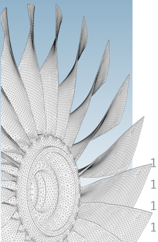
[127] A. M. Panunzio et al. en. American Society of Mechanical Engineers Digital Collection, 2015. DOI: 10.1115/GT2015-43560.

[128] C. Joannin et al. en. *Journal of Engineering for Gas Turbines and Power* (2016). DOI: 10.1115/1.4031886.



Contact mechanics

Principle

- numerical management of unilateral contact constraints (with or without friction) remains a very active field of research[129]
- contact implies both discontinuous...
  - ...velocity[130] and...
  - ...acceleration fields
- a mechanical system with contact interfaces is inherently nonlinear
  - it is difficult to accurately predict its eigenfrequencies[131]
  - uncertainty on the system's life span[132]

Specificity of blade/casing interface

- very high relative speeds (500 m/s)
- very small incursion depth
- use of industrial finite elements (quadratic elements)

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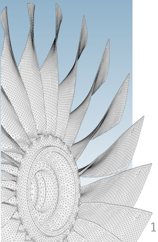
[129] A. Thorin et al. en. *Journal of Engineering for Gas Turbines and Power* (2019). DOI: 10.1115/1.4040857.

[130] M. B. Meingast et al. en. *International Journal of Non-Linear Mechanics* (2014). DOI: 10.1016/j.ijnonlinmec.2014.01.007.

[131] D. Laxalde et al. en. *Computational Mechanics* (2011). DOI: 10.1007/s00466-010-0556-3.

[132] W. Feifei et al. en. *Procedia Engineering* (2015). DOI: 10.1016/j.proeng.2014.12.661.



Contact mechanics

Solution algorithms

- the suitability of finite element models for contact analyses is questioned[133]
- solving a contact problem involves the computation of two key quantities:
  - the gap (blade/casing clearance)
  - contact forces

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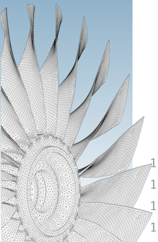
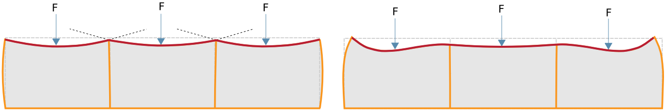
[133] C. Yoong et al. en. *Nonlinear Dynamics* (2018). DOI: 10.1007/s11071-017-4025-9.



Contact mechanics

Solution algorithms

- **gap:** a node-to-surface strategy is usually employed [134]
  - quadratic FE on the casing typically yields a discontinuous normal direction to the contact surface
  - smoothing techniques are thus required for flexible casings



quadratic elements (industry standard) use of Hermite elements

  - challenge: compatibility with industrial procedures
  - Hermite or mortar elements (mesh modification) [135]
  - use of non-intrusive B-splines or NURBS [136]
  - isogeometric analysis [137]

⇒ significant numerical cost but robust procedures and industry ready solutions are available

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[134] P. Wriggers. en. 2nd edition. Berlin Heidelberg: Springer-Verlag, 2006. DOI: 10.1007/978-3-540-32609-0.

[135] K. A. Fischer et al. en. *Computer Methods in Applied Mechanics and Engineering* (2006). DOI: 10.1016/j.cma.2005.09.025.

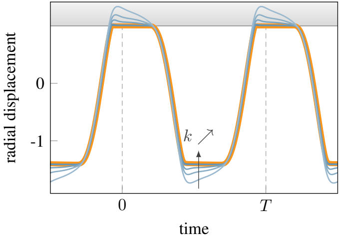
[136] T. Belytschko et al. en. *International Journal for Numerical Methods in Engineering* (2002). DOI: 10.1002/nme.568.

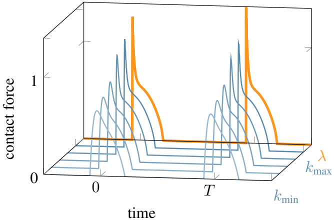
[137] T. J. R. Hughes et al. en. *Computer Methods in Applied Mechanics and Engineering* (2005). DOI: 10.1016/j.cma.2004.10.008.

Contact mechanics

Solution algorithms

- **contact forces:**
  - penalty method, numerical parameter k [138]
  - Lagrange multiplier-based approaches [139]





- 1-dof system with rigid impacted surface: $k \nearrow \rightarrow \lambda$
- numerical sensitivity increases with $k \Rightarrow$ need for smaller time steps
- k is often times used as a representation of casing stiffness [140]
- Lagrange multipliers-based strategies inherently forbid penetrations
- numerical artefacts on contact forces with Lagrange multipliers (initial impulsion = $f(\delta t)$)

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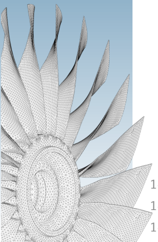
138. H. Ma et al. en. *Journal of Sound and Vibration* (2015). DOI: 10.1016/j.jsv.2014.10.020.
139. N. J. Carpenter et al. en. *International Journal for Numerical Methods in Engineering* (1991). DOI: 10.1002/nme.1620320107.
140. H. Ma et al. en. *Journal of Vibration and Control* (2015). DOI: 10.1177/1077546315575835.

Contact mechanics

Solution algorithms

- **contact forces:**
  - penalty method, numerical parameter k
  - Lagrange multiplier-based approaches [141]
  - θ-method [142]
  - linear complementarity problem formulation [143]
  - ...



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141. N. J. Carpenter et al. en. *International Journal for Numerical Methods in Engineering* (1991). DOI: 10.1002/nme.1620320107.

142. A. Thorin et al. en. *Journal of Engineering for Gas Turbines and Power* (2019). DOI: 10.1115/1.4040857.

143. M. B. Meingast et al. en. *International Journal of Non-Linear Mechanics* (2014). DOI: 10.1016/j.ijnonlinmec.2014.01.007.

Contact mechanics

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- contact forces:
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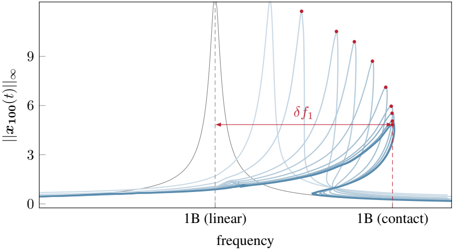
  - Lagrange multiplier-based approaches[141]
  - θ-method[142]
  - linear complementarity problem formulation[143]
  - ...
  - critical issue for design purposes: modelling of contact stiffening



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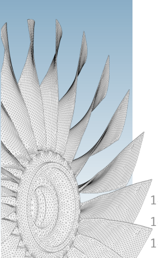
141. N. J. Carpenter et al. en. *International Journal for Numerical Methods in Engineering* (1991). DOI: `10.1002/nme.1620320107`.

142. A. Thorin et al. en. *Journal of Engineering for Gas Turbines and Power* (2019). DOI: `10.1115/1.4040857`.

143. M. B. Meingast et al. en. *International Journal of Non-Linear Mechanics* (2014). DOI: `10.1016/j.ijnonlinmec.2014.01.007`.



Contact mechanics

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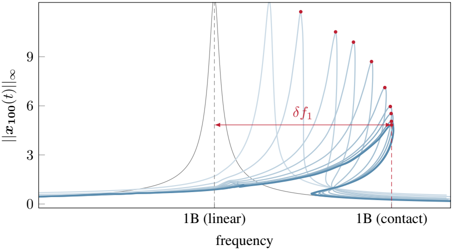

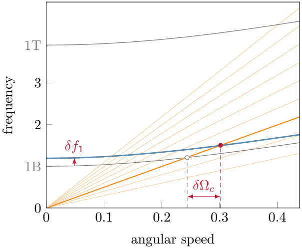

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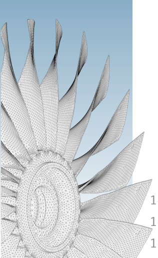
[141] N. J. Carpenter et al. en. *International Journal for Numerical Methods in Engineering* (1991). DOI: 10.1002/nme.1620320107.

[142] A. Thorin et al. en. *Journal of Engineering for Gas Turbines and Power* (2019). DOI: 10.1115/1.4040857.

[143] M. B. Meingast et al. en. *International Journal of Non-Linear Mechanics* (2014). DOI: 10.1016/j.ijnonlinmec.2014.01.007.



Contact mechanics

Solution algorithms

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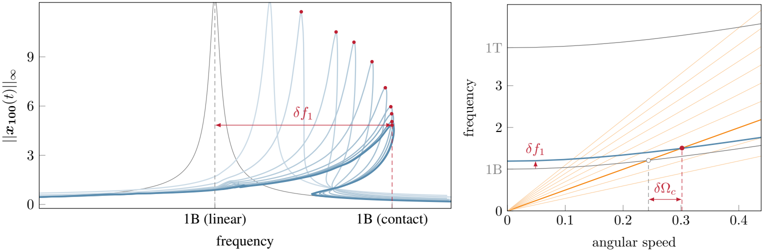

⟹ significant impact on the prediction of critical speeds

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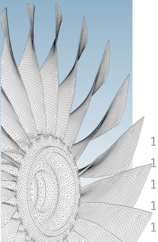
141. N. J. Carpenter et al. en. *International Journal for Numerical Methods in Engineering* (1991). DOI: 10.1002/nme.1620320107.
142. A. Thorin et al. en. *Journal of Engineering for Gas Turbines and Power* (2019). DOI: 10.1115/1.4040857.
143. M. B. Meingast et al. en. *International Journal of Non-Linear Mechanics* (2014). DOI: 10.1016/j.ijnonlinmec.2014.01.007.



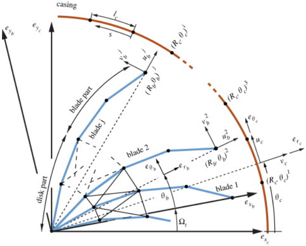
Modelling of the system

Types of models

- analytical models[144]
- simplified planar models[145]
- simplified 3D models[146]
- full industrial 3D models[147]

Simplified models are usually employed for:

- whole engine dynamics prediction (shaft motions)
- development and validation of new methodologies



- 2D planar model of a fan stage[146]

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144. S. K. Sinha. en. *International Journal of Non-Linear Mechanics* (2005). DOI: 10.1016/j.ijnonlinmec.2004.05.019.
145. M. Legrand et al. en. *Journal of Sound and Vibration* (2009). DOI: 10.1016/j.jsv.2008.06.019.
146. H. Ma et al. en. *Journal of Vibration and Control* (2015). DOI: 10.1177/1077546315575835.
147. N. Guérin et al. en. *Journal of Engineering for Gas Turbines and Power* (2019). DOI: 10.1115/1.4040858.
148. A. Batailly et al. en. *Journal of Sound and Vibration* (2015). DOI: 10.1016/j.jsv.2015.03.027.

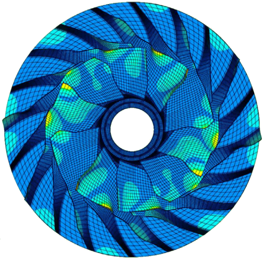
Modelling of the system

Types of models

- analytical models[144]
- simplified planar models[145]
- simplified 3D models[146]
- full industrial 3D models[147]

Accurate predictions of the vibration response of bladed disks require full 3D models:

- key for accurate stress levels assessment
- high computational cost ($\sim 10^6$ dof)
- model reduction is required



▶ 3D model of an impeller[148]

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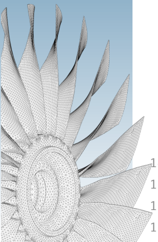
[144] S. K. Sinha. en. *International Journal of Non-Linear Mechanics* (2005). DOI: 10.1016/j.ijnonlinmec.2004.05.019.

[145] M. Legrand et al. en. *Journal of Sound and Vibration* (2009). DOI: 10.1016/j.jsv.2008.06.019.

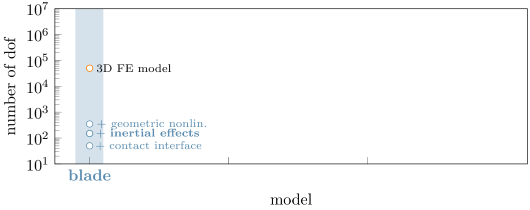
[146] H. Ma et al. en. *Journal of Vibration and Control* (2015). DOI: 10.1177/1077546315575835.

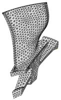
[147] N. Guérin et al. en. *Journal of Engineering for Gas Turbines and Power* (2019). DOI: 10.1115/1.4040858.

[148] A. Batailly et al. en. *Journal of Sound and Vibration* (2015). DOI: 10.1016/j.jsv.2015.03.027.



Model reduction





- industrial 3D FE models $(N$ dof$)$ required for an accurate representation of the contact interface
- numerical efficiency ⇒ CMS techniques $(n << N$ dof$)$
- fixed-interface methods are usually preferred for numerical stability [149]
- centrifugal stiffening may be accounted for [150] $(n \to 3n$ dof$)$
- recent developments (friction modelling and geometric nonlinearities) [151, 152]

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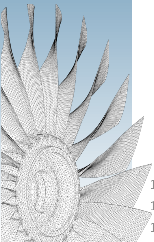
[149] R. Bladh et al. *AIAA Journal* (2003). DOI: 10.2514/2.2123.

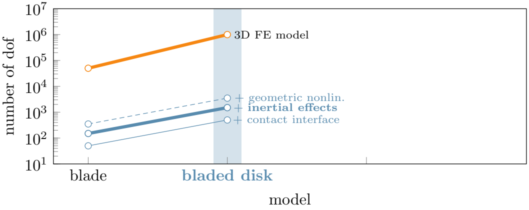
[150] A. Sternchuss et al. Louvain, 2006, available online.

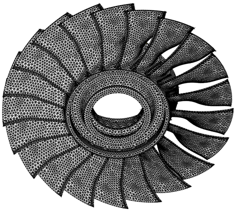
[151] M. Balmaseda et al. en. American Society of Mechanical Engineers Digital Collection, 2019. DOI: 10.1115/GT2019-90813.

[152] E. P. Petrov. en. *Journal of Engineering for Gas Turbines and Power* (2019). DOI: 10.1115/1.4045183.



Model reduction





- property of cyclic symmetry ⇒ efficient computation of reduced order models $(10n+$ dof)
- inertial effects may be accounted for [153] $(30n+$ dof)
- sensitive numerical model due to spurious modes and damping quantification
- recent developments include the combination of mistuning and contact interfaces [154, 155]
- limited published experimental results involving disk modes have limited the use of such models

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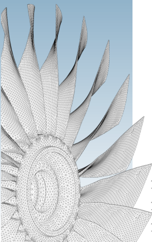
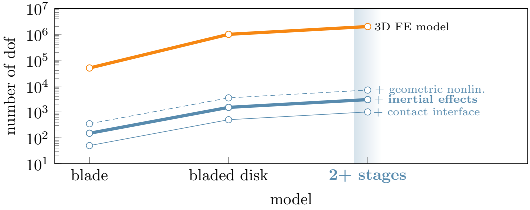
[153] J. Paltrinieri et al. en. American Society of Mechanical Engineers Digital Collection, 2017. DOI: 10.1115/GT2017-63488.

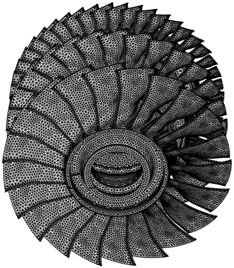
[154] C. Joannin et al. en. *Journal of Engineering for Gas Turbines and Power* (2016). DOI: 10.1115/1.4031886.

[155] J. Joachim. fr. PhD thesis. Ecole Polytechnique de Montréal, 2020, OAI: tel-02888893.



Model reduction





- multi-stage cyclic symmetry[156, 157] is the foundation for such numerical models
- inter-stage coupling strategies have also been developed[158]
- high numerical cost when combined with time or frequency methods for nonlinear dynamics
- current bottleneck as accurate modelling of the component dynamics is required relevant numerical predictions

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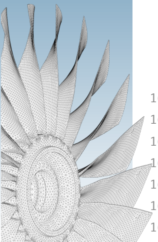
[156] D. Laxalde et al. en. *Computers & Structures* (2011). DOI: 10.1016/j.compstruc.2010.10.020.

[157] D.-M. Tran. en. *Journal of Sound and Vibration* (2014). DOI: 10.1016/j.jsv.2014.06.004.

[158] K. D'Souza et al. *AIAA Journal* (2012). DOI: 10.2514/1.J051021.



Multiphysics simulations

Blade/casing interactions yield intrinsically multiphysics systems

- vibration of the rotor (and the stator)
- wear of the abradable coating[159, 160, 161]
- aerodynamic loading in engine conditions
  - interactions experimentally observed under vacuum[162]
  - when accounted for, simplified loadings[163]
- thermomechanics[164, 165](extreme temperature in turbine stages)

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160. **M. Legrand et al.** en. *Journal of Computational and Nonlinear Dynamics* (2012). DOI: 10.1115/1.4004951.

161. **R. J. Williams.** en. American Society of Mechanical Engineers Digital Collection, 2012. DOI: 10.1115/GT2011-45495.

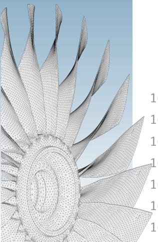
162. **P. Almeida et al.** en. *Journal of Engineering for Gas Turbines and Power* (2016). DOI: 10.1115/1.4033065.

163. **A. Millecamps et al.** en. American Society of Mechanical Engineers Digital Collection, 2010. DOI: 10.1115/DETC2009-86842.

164. **A. Batailly et al.** en. American Society of Mechanical Engineers Digital Collection, 2014. DOI: 10.1115/GT2014-25675.

165. **Q. Agrapart et al.** en. *Journal of Sound and Vibration* (2019). DOI: 10.1016/j.jsv.2019.114869.

166. **F. Nyssen et al.** en. *Journal of Engineering for Gas Turbines and Power* (2019). DOI: 10.1115/1.4041647.



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⟹ mostly a computational challenge

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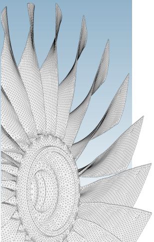
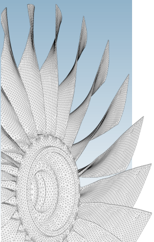
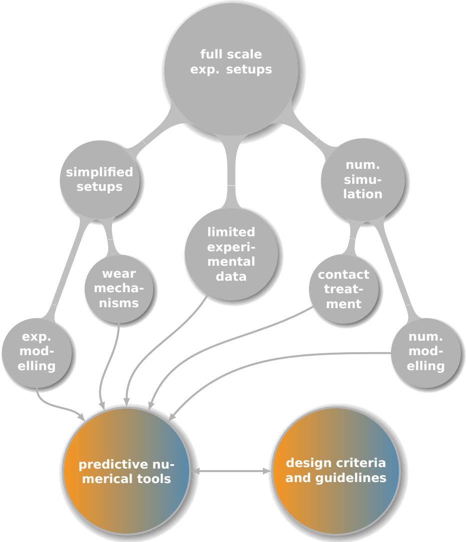
160. **M. Legrand et al.** en. *Journal of Computational and Nonlinear Dynamics* (2012). DOI: 10.1115/1.4004951.
161. **R. J. Williams.** en. American Society of Mechanical Engineers Digital Collection, 2012. DOI: 10.1115/GT2011-45495.
162. **P. Almeida et al.** en. *Journal of Engineering for Gas Turbines and Power* (2016). DOI: 10.1115/1.4033065.
163. **A. Millecamps et al.** en. American Society of Mechanical Engineers Digital Collection, 2010. DOI: 10.1115/DETC2009-86842.
164. **A. Batailly et al.** en. American Society of Mechanical Engineers Digital Collection, 2014. DOI: 10.1115/GT2014-25675.
165. **Q. Agrapart et al.** en. *Journal of Sound and Vibration* (2019). DOI: 10.1016/j.jsv.2019.114869.
166. **F. Nyssen et al.** en. *Journal of Engineering for Gas Turbines and Power* (2019). DOI: 10.1115/1.4041647.



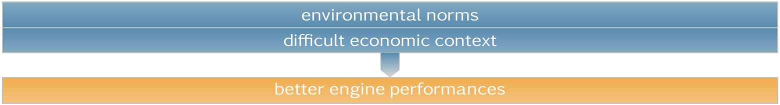
③ Industrial applications





full scale exp. setups
simplified setups
num. simulation
limited experimental data
wear mechanisms
contact treatment
exp. modelling
num. modelling
predictive numerical tools
design criteria and guidelines

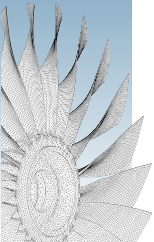
Industrial context: today

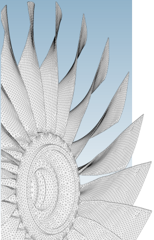


environmental norms

difficult economic context

better engine performances





Industrial context: today

environmental norms

difficult economic context

better engine performances

new lighter materials

reduced operating clearances

Industrial context: today

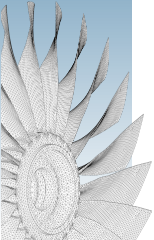
environmental norms

difficult economic context

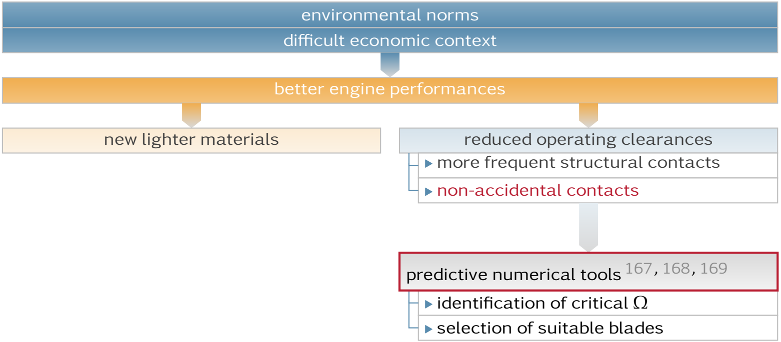
better engine performances

new lighter materials

reduced operating clearances
- more frequent structural contacts
- non-accidental contacts



Industrial context: today



environmental norms

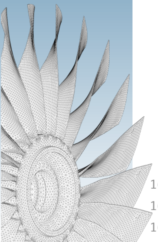
difficult economic context

better engine performances

new lighter materials

reduced operating clearances
- more frequent structural contacts
- non-accidental contacts

predictive numerical tools[167, 168, 169]
- identification of critical Ω
- selection of suitable blades



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[167] E. P. Petrov et al. en. *Journal of Turbomachinery* (2004). DOI: 10.1115/1.1644557.

[168] F. El Haddad et al. Glasgow, Scotland, 2018, online reference.

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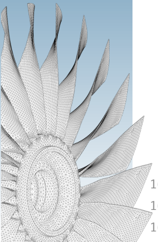
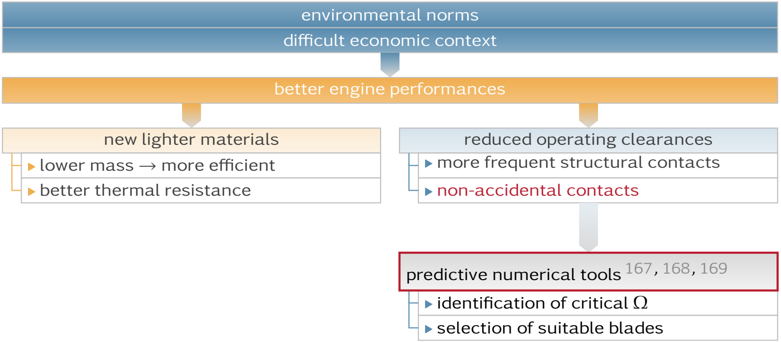


Industrial context: today



environmental norms

difficult economic context

better engine performances

new lighter materials

- lower mass → more efficient
- better thermal resistance

reduced operating clearances

- more frequent structural contacts
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predictive numerical tools [167, 168, 169]

- identification of critical Ω
- selection of suitable blades

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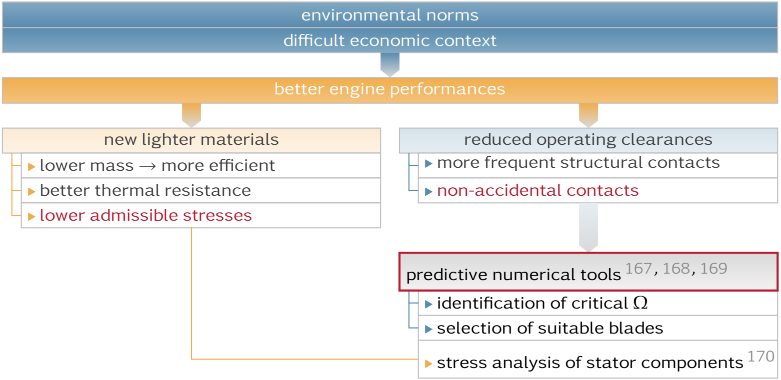

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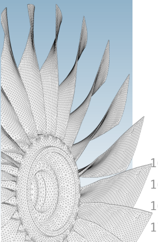

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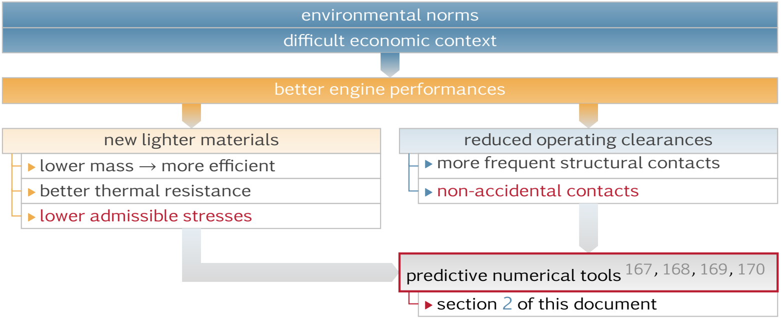
Industrial context: today



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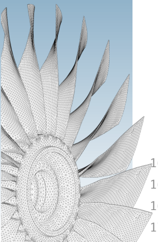

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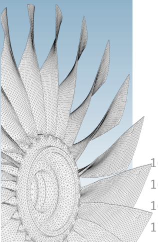

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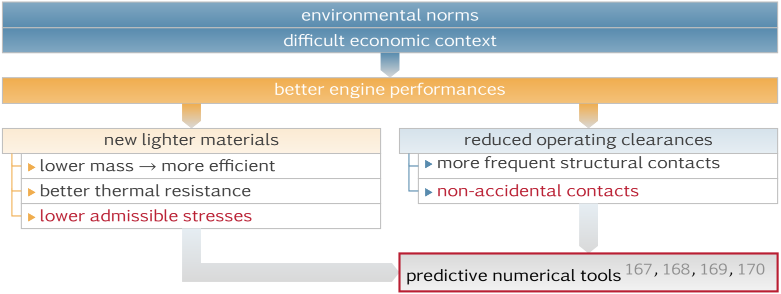

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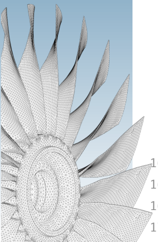

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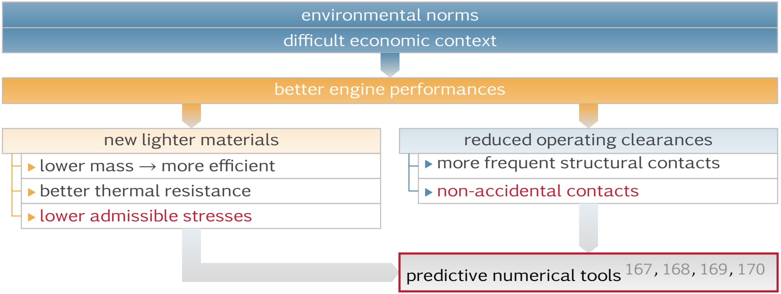

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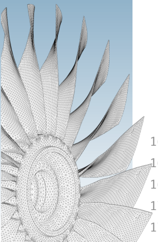

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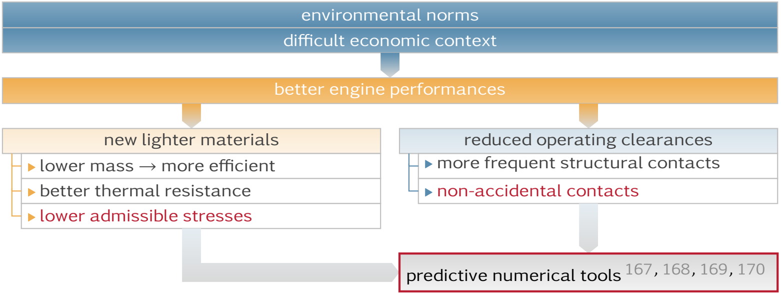

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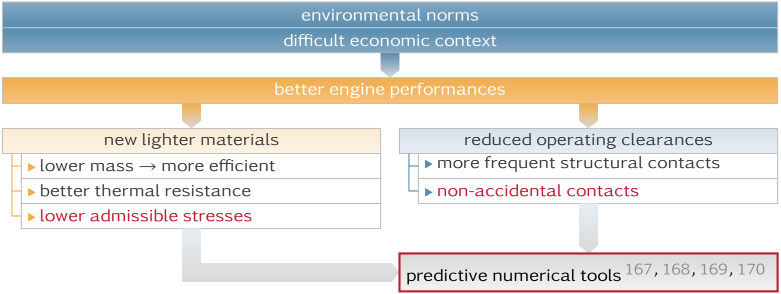

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② in progress: account for contact during blades design stage (constraint) [171]

③ future: define new design criteria and guidelines for robust blades with respect to contact interactions

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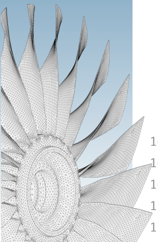
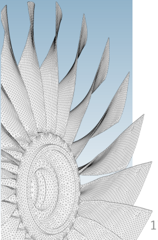


Accounting for contact in blade design

Current design strategies

- emphasis on aerodynamic performances
- need for minimal operating clearances
- recent design recommendations promote blade profiles that reduce operating clearances as they vibrate [172]
- nonlinear dynamics considerations may only be accounted for *a posteriori*

⇒ significant cost as existing predictive tools can only be used to discriminate existing blade profiles based on their response to contact



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[172] E. Erler et al. en. *Journal of Turbomachinery* (2016). DOI: 10.1115/1.4031865.

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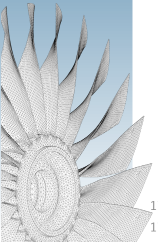

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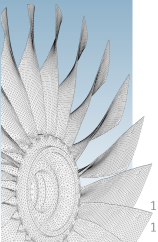
⇒ opposition between aerodynamics and nonlinear dynamics considerations [172, 173]

⇒ contact must be accounted for early in the blade design stage



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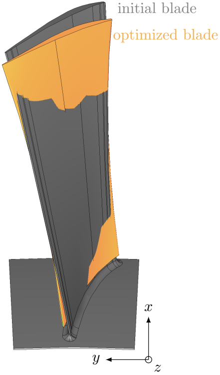
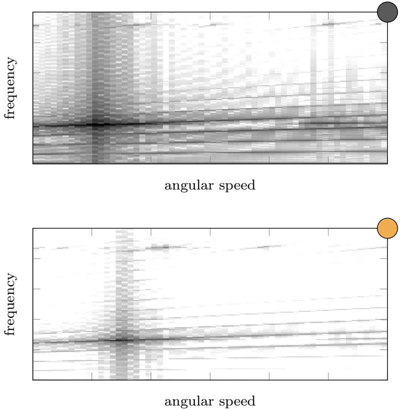
173. A. Batailly et al. en. *Journal of Engineering for Gas Turbines and Power* (2015). DOI: 10.1115/1.4028263.

Accounting for contact in blade design

Development of automated optimization procedures

- based on existing design criteria [174]
- reverse engineering of existing blade profiles and optimization





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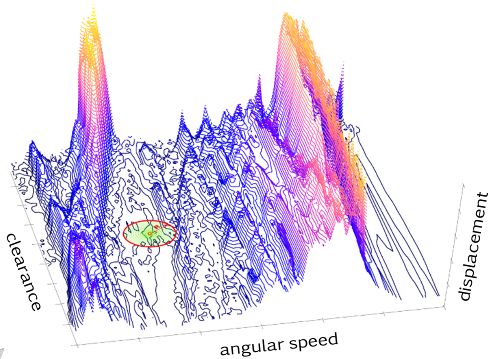
[174] A. Batailly et al. en. American Society of Mechanical Engineers Digital Collection, 2016. DOI: 10.1115/GT2016-56721.

Accounting for contact in blade design

Development of new optimization algorithms

- new optimization techniques are required for nonsmooth objective functions [175]
- optimal design points must be found *far enough* from potential bifurcations in the space of variables
- blackbox optimization framework well-suited for costly industrial procedures



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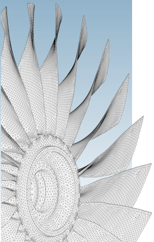
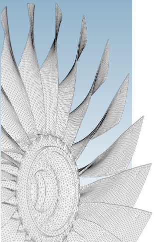
[175] C. Audet et al. *SIAM Journal on Optimization* (2020, submitted).



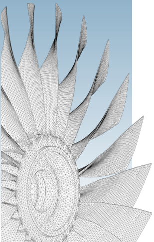
④ Going forward...



On-going research and future directions

Several open questions

- understanding of complex wear mechanisms
- better understand the physics of the phenomena (thermomechanics)
- development of more efficient numerical solution strategies
- use of high fidelity industrial multi-stage models
- assess the robustness of numerical predictions to uncertainties



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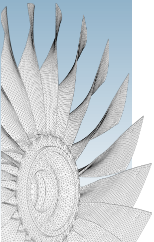
One major road block

- accurate numerical predictions require test data ⇒ very few available
- selection of a suitable wear law requires test data ⇒ mostly confidential
- characterization of new lighter materials is limited due to confidentiality
- components design itself is often confidential



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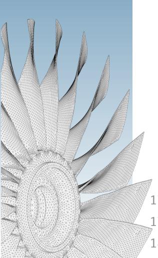
⇒ it is, most of the time, **impossible to reproduce published results**



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Promoting data exchange and numerical comparisons

- publications as open archives (or in open access journals) are not the only solution

  ASME DIGITAL COLLECTION HAL JTCAM

- promote the dissemination of numerical tools [176, 177] ⇒ open source

  Software Heritage platform

- use of standardized test cases [178] (see CFD code validations in the 1990s) ⇒ open blade models

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178. E. Piollet et al. en. *Journal of Sound and Vibration* (2019). DOI: 10.1016/j.jsv.2019.114878.
179. M. Krack et al. en. Cham: Springer International Publishing, 2019. DOI: 10.1007/978-3-030-14023-6.
180. E. Piollet et al. 2019, code available online.

Thank you for your attention!

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https://lava-wiki.meca.polymtl.ca/te2020/home

Bibliographic references I

Agrapart, Q. et al. « Multi-physics numerical simulation of an experimentally predicted rubbing event in aircraft engines ». en. *Journal of Sound and Vibration* 460 (Nov. 2019), page 114869. ISSN: 0022-460X. DOI: 10.1016/j.jsv.2019.114869.

Ahmad, S. « Rotor Casing Contact Phenomenon in Rotor Dynamics — Literature Survey ». *Journal of Vibration and Control* 16.9 (2010), pages 1369–1377. DOI: 10.1177/1077546309341605.

Almeida, P. et al. « Numerical Analysis of Bladed Disk–Casing Contact With Friction and Wear ». en. *Journal of Engineering for Gas Turbines and Power* 138.12 (Dec. 2016). ISSN: 0742-4795. DOI: 10.1115/1.4033065.

Audet, C. et al. « Escaping unknown discontinuous regions in blackbox optimization ». *SIAM Journal on Optimization* submitted (2020, submitted).

Balmaseda, M. et al. « Reduced Order Models for Nonlinear Dynamic Analysis With Application to a Fan Blade ». en. American Society of Mechanical Engineers Digital Collection, Nov. 2019. DOI: 10.1115/GT2019-90813.

Batailly, A. et al. « Experimental and numerical simulation of a rotor/stator interaction event localized on a single blade within an industrial high-pressure compressor ». en. *Journal of Sound and Vibration* 375 (Aug. 2016), pages 308–331. ISSN: 0022-460X. DOI: 10.1016/j.jsv.2016.03.016.

Batailly, A. et al. « High-Pressure Compressor Blade Dynamics Under Aerodynamic and Blade-Tip Unilateral Contact Forcings ». en. American Society of Mechanical Engineers Digital Collection, Sept. 2014. DOI: 10.1115/GT2014-25675.

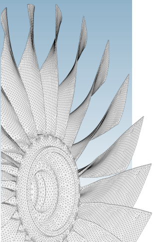
— .« Numerical-Experimental Comparison in the Simulation of Rotor/Stator Interaction Through Blade-Tip/Abradable Coating Contact ». en. *Journal of Engineering for Gas Turbines and Power* 134.8 (Aug. 2012). ISSN: 0742-4795. DOI: 10.1115/1.4006446.

Batailly, A. et al. « Redesign of a High-Pressure Compressor Blade Accounting for Nonlinear Structural Interactions ». en. *Journal of Engineering for Gas Turbines and Power* 137.2 (Feb. 2015). Publisher: American Society of Mechanical Engineers Digital Collection. ISSN: 0742-4795. DOI: 10.1115/1.4028263.

Batailly, A. et al. « Unilateral contact induced blade/casing vibratory interactions in impellers: Analysis for flexible casings with friction and abradable coating ». en. *Journal of Sound and Vibration* 348 (July 2015), pages 344–364. ISSN: 0022-460X. DOI: 10.1016/j.jsv.2015.03.027.

Batailly, A. et al. « Minimising Clearance Consumption: A Key Factor for the Design of Blades Robust to Rotor/Stator Interactions? » en. American Society of Mechanical Engineers Digital Collection, Sept. 2016. DOI: 10.1115/GT2016-56721.

Battiato, G. et al. « Reduced Order Modeling for Multistage Bladed Disks With Friction Contacts at the Flange Joint ». en. *Journal of Engineering for Gas Turbines and Power* 140.5 (May 2018). Publisher: American Society of Mechanical Engineers Digital Collection. ISSN: 0742-4795. DOI: 10.1115/1.4038348.



Bibliographic references II

Belytschko, T. et al. « A monolithic smoothing-gap algorithm for contact-impact based on the signed distance function ». en. *International Journal for Numerical Methods in Engineering* 55.1 (2002). _eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1002/nme.568, pages 101–125. ISSN: 1097-0207. DOI: 10.1002/nme.568.

Berthoul, B. et al. « Phenomenological modeling of abradable wear in turbomachines ». en. *Mechanical Systems and Signal Processing* 98 (Jan. 2018), pages 770–785. ISSN: 0888-3270. DOI: 10.1016/j.ymssp.2017.05.020.

Bill, R. C. et al. *Friction and Wear of Several Compressor Gas-Path Seal Materials.* en. Jan. 1978, available online.

Bill, R. C. et al. « Friction and Wear of Sintered Fibermetal Abradable Seal Materials ». en. *Journal of Lubrication Technology* 99.4 (Oct. 1977). Publisher: American Society of Mechanical Engineers Digital Collection, pages 421–427. ISSN: 0022-2305. DOI: 10.1115/1.3453236.

Black, H. F. « Interaction of a Whirling Rotor with a Vibrating Stator across a Clearance Annulus: » en. *Journal of Mechanical Engineering Science* (Feb. 2006). Publisher: SAGE PublicationsSage UK: London, England. DOI: 10.1243/JMES_JOUR_1968_010_003_02.

Bladh, R. et al. « Numerical Instability of Classical Free-Interface Component Mode Synthesis Techniques ». *AIAA Journal* 41.8 (2003). Publisher: American Institute of Aeronautics and Astronautics _eprint: https://doi.org/10.2514/2.2123, pages 1621–1624. ISSN: 0001-1452. DOI: 10.2514/2.2123.

Borel, M. O. et al. « The wear mechanisms occurring in abradable seals of gas turbines ». en. *Surface and Coatings Technology* 39-40 (Dec. 1989), pages 117–126. ISSN: 0257-8972. DOI: 10.1016/0257-8972(89)90046-7.

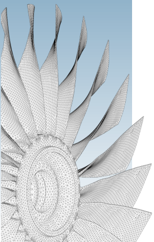
Bounazef, M. et al. « The wear, deterioration and transformation phenomena of abradable coating BN–SiAl–bounding organic element, caused by the friction between the blades and the turbine casing ». en. *Materials Letters* 58.27 (Nov. 2004), pages 3375–3380. ISSN: 0167-577X. DOI: 10.1016/j.matlet.2004.02.049.

Carpenter, N. J. et al. « Lagrange constraints for transient finite element surface contact ». en. *International Journal for Numerical Methods in Engineering* 32.1 (1991), pages 103–128. ISSN: 1097-0207. DOI: 10.1002/nme.1620320107.

Childs, D. W. « A Modal Transient Rotordynamic Model for Dual-Rotor Jet Engine Systems ». en. *Journal of Engineering for Industry* 98.3 (Aug. 1976). Publisher: American Society of Mechanical Engineers Digital Collection, pages 876–882. ISSN: 0022-0817. DOI: 10.1115/1.3439046.

— .« Rub-Induced Parametric Excitation in Rotors ». en. *Journal of Mechanical Design* 101.4 (Oct. 1979). Publisher: American Society of Mechanical Engineers Digital Collection, pages 640–644. ISSN: 0161-8458. DOI: 10.1115/1.3454114.

Christie, C. A. *Task 14 - Engine windmilling imbalance loads.* Aviation Rulemaking Advisory Committee FR Doc. 96-16960. FAA, 1996, available online.



Bibliographic references III

Clingman, D. L. S. « Material removal considerations for metal-ceramic abradable turbine seal systems ». *Lubrication Engineering* (Nov. 1983, available online), p. 712–716.

Colón, S. et al. « Geometric Effects of Thermal Barrier Coating Damage on Turbine Blade Temperatures ». en. American Society of Mechanical Engineers Digital Collection, Nov. 2019. DOI: `10.1115/GT2019-90886`.

Cuny, M. et al. « Experimental Measurement of Dynamic Forces Generated during Short-Duration Contacts: Application to Blade-Casing Interactions in Aircraft Engines ». en. *Experimental Mechanics* 54.2 (Feb. 2014), pages 101–114. ISSN: 1741-2765. DOI: `10.1007/s11340-013-9780-z`.

Dadouche, A. et al. « Experimental Evaluation of Abradable Seal Performance at High Temperature ». en. American Society of Mechanical Engineers Digital Collection, Aug. 2009, pages 143–150. DOI: `10.1115/GT2008-51228`.

Delebarre, C. et al. « An experimental study of the high speed interaction between a labyrinth seal and an abradable coating in a turbo-engine application ». en. *Wear* 316.1 (Aug. 2014), pages 109–118. ISSN: 0043-1648. DOI: `10.1016/j.wear.2014.04.023`.

Dimarogonas, A. D. et al. « Packing rub effect in rotating machinery. I. A state of the art review ». en. *Wear* 14.3 (Sept. 1969), pages 153–170. ISSN: 0043-1648. DOI: `10.1016/0043-1648(69)90037-4`.

Dring, R. P. et al. « Turbine Rotor-Stator Interaction ». en. *Journal of Engineering for Power* 104.4 (Oct. 1982). Publisher: American Society of Mechanical Engineers Digital Collection, pages 729–742. ISSN: 0022-0825. DOI: `10.1115/1.3227339`.

D'Souza, K. et al. « Reduced-Order Modeling for Nonlinear Analysis of Cracked Mistuned Multistage Bladed-Disk Systems ». *AIAA Journal* 50.2 (2012). Publisher: American Institute of Aeronautics and Astronautics _eprint: https://doi.org/10.2514/1.J051021, pages 304–312. ISSN: 0001-1452. DOI: `10.2514/1.J051021`.

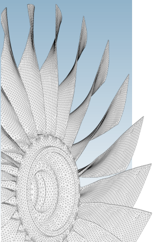
Edwards, S. et al. « The Influence of Torsion on Rotor/Stator Contact in Rotating Machinery ». en. *Journal of Sound and Vibration* 225.4 (Aug. 1999), pages 767–778. ISSN: 0022-460X. DOI: `10.1006/jsvi.1999.2302`.

Ehrich, F. F. « High Order Subharmonic Response of High Speed Rotors in Bearing Clearance ». en. *Journal of Vibration, Acoustics, Stress, and Reliability in Design* 110.1 (Jan. 1988). Publisher: American Society of Mechanical Engineers Digital Collection, pages 9–16. ISSN: 0739-3717. DOI: `10.1115/1.3269488`.

El Haddad, F. et al. « Object-Oriented Fortran to model Non-Linear Vibration ». *6th European Conference on Computational Mechanics ECCM 6*. Glasgow, Scotland, 2018, online reference.

Emery, A. F. et al. « An experimental investigation of temperatures due to rubbing at the blade-seal interface in an aircraft compressor ». en. *Wear* 91.2 (Nov. 1983), pages 117–130. ISSN: 0043-1648. DOI: `10.1016/0043-1648(83)90248-X`.

Erler, E. et al. « Desensitization of Axial Compressor Performance and Stability to Tip Clearance Size ». en. *Journal of Turbomachinery* 138.3 (Mar. 2016). ISSN: 0889-504X. DOI: `10.1115/1.4031865`.



Bibliographic references IV

Faraoun, H. I. et al. « Modelling route for abradable coatings ». en. *Surface and Coatings Technology* 200.22 (June 2006), pages 6578–6582. ISSN: 0257-8972. DOI: 10.1016/j.surfcoat.2005.11.105.

Feifei, W. et al. « Failure Analysis of Rubbing of the Fan Tip and Case of an Engine ». en. *Procedia Engineering*. 2014 Asia-Pacific International Symposium on Aerospace Technology, APISAT2014 September 24-26, 2014 Shanghai, China 99 (Jan. 2015), pages 1289–1296. ISSN: 1877-7058. DOI: 10.1016/j.proeng.2014.12.661.

Fischer, K. A. et al. « Mortar based frictional contact formulation for higher order interpolations using the moving friction cone ». en. *Computer Methods in Applied Mechanics and Engineering*. John H. Argyris Memorial Issue. Part I 195.37 (July 2006), pages 5020–5036. ISSN: 0045-7825. DOI: 10.1016/j.cma.2005.09.025.

Gallus, H. E. « Experimental Methods and Results in Unsteady Aerodynamics and Aeroelasticity ». en. *Unsteady Aerodynamics, Aeroacoustics, and Aeroelasticity of Turbomachines and Propellers*. New York, NY: Springer, 1993, pages 455–485. ISBN: 978-1-4613-9341-2. DOI: 10.1007/978-1-4613-9341-2_23.

Groll, G. von et al. « The Harmonic Balance Method with Arc-Length Continuation in Rotor/Stator Contact Problems ». en. *Journal of Sound and Vibration* 241.2 (Mar. 2001), pages 223–233. ISSN: 0022-460X. DOI: 10.1006/jsvi.2000.3298.

Guérin, N. et al. « Thermomechanical Model Reduction for Efficient Simulations of Rotor-Stator Contact Interaction ». en. *Journal of Engineering for Gas Turbines and Power* 141.2 (Feb. 2019). ISSN: 0742-4795. DOI: 10.1115/1.4040858.

He, L. et al. « Three-Dimensional Time-Marching Inviscid and Viscous Solutions for Unsteady Flows Around Vibrating Blades ». en. *Journal of Turbomachinery* 116.3 (July 1994). Publisher: American Society of Mechanical Engineers Digital Collection, pages 469–476. ISSN: 0889-504X. DOI: 10.1115/1.2929436.

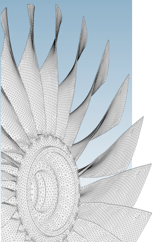
Hughes, T. J. R. et al. « Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement ». en. *Computer Methods in Applied Mechanics and Engineering* 194.39 (Oct. 2005), pages 4135–4195. ISSN: 0045-7825. DOI: 10.1016/j.cma.2004.10.008.

Hulme, C. J. et al. « Axial Compressor Blade Failure, Design Modification, and its Validation ». en. American Society of Mechanical Engineers Digital Collection, Aug. 2015. DOI: 10.1115/GT2015-43312.

Investigation: 200700356 - In-flight engine failure, Sydney, 03 February 2007, Boeing Company 747-438, VH-OJM. Technical report. Australian Transport Safety Bureau, 2008, available online.

Isomura, K. et al. « A Numerical Study of Flutter in a Transonic Fan ». en. *Journal of Turbomachinery* 120.3 (July 1998). Publisher: American Society of Mechanical Engineers Digital Collection, pages 500–507. ISSN: 0889-504X. DOI: 10.1115/1.2841746.

Jacquet-Richardet, G. et al. « Rotor to stator contacts in turbomachines. Review and application ». en. *Mechanical Systems and Signal Processing* 40.2 (Nov. 2013), pages 401–420. ISSN: 0888-3270. DOI: 10.1016/j.ymssp.2013.05.010.



Bibliographic references V

Joachim, J. « Développement de modèles numériques de roues aubagées désaccordées dans un contexte non-linéaire ». fr. PhD thesis. Ecole Polytechnique de Montréal, June 2020, OAI: tel-02888893.

Joachim, J. et al. « Numerical Investigation of a Mistuned Academic Bladed Disk Dynamics with Blade/Casing Contact ». en. *Journal of Engineering for Gas Turbines and Power* (2020). DOI: 10.1115/1.4047780.

Joannin, C. et al. « Nonlinear Modal Analysis of Mistuned Periodic Structures Subjected to Dry Friction ». en. *Journal of Engineering for Gas Turbines and Power* 138.7 (July 2016). Publisher: American Society of Mechanical Engineers Digital Collection. ISSN: 0742-4795. DOI: 10.1115/1.4031886.

John, H. R. et al. *Aircraft accident report: national Airlines, Inc. DC-10-10, N60NA, near Albuquerque, New Mexico*. Technical report NTSB-AAR-75-2. National Transport Safety Board, 1975, available online.

Jones, S. et al. « Forced vibrations of a turbine blade undergoing regularized unilateral contact conditions through the wavelet balance method ». en. *International Journal for Numerical Methods in Engineering* 101.5 (2015). _eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1002/nme.4807, pages 351–374. ISSN: 1097-0207. DOI: 10.1002/nme.4807.

Kemp, N. H. et al. « Aerodynamic Interference Between Moving Blade Rows ». *Journal of the Aeronautical Sciences* 20.9 (1953). Publisher: American Institute of Aeronautics and Astronautics _eprint: https://doi.org/10.2514/8.2758, pages 585–597. DOI: 10.2514/8.2758.

Kerschen, G. et al. « Nonlinear normal modes, Part I: A useful framework for the structural dynamicist ». en. *Mechanical Systems and Signal Processing*. Special Issue: Non-linear Structural Dynamics 23.1 (Jan. 2009), pages 170–194. ISSN: 0888-3270. DOI: 10.1016/j.ymssp.2008.04.002.

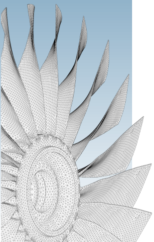
Krack, M. et al. « Introduction to Harmonic Balance and application to nonlinear vibrations ». en. American Society of Mechanical Engineers Digital Collection, 2018.

Krack, M. « Nonlinear modal analysis of nonconservative systems: Extension of the periodic motion concept ». en. *Computers & Structures* 154 (July 2015), pages 59–71. ISSN: 0045-7949. DOI: 10.1016/j.compstruc.2015.03.008.

Krack, M. et al. *Harmonic Balance for Nonlinear Vibration Problems*. en. Mathematical Engineering. Cham: Springer International Publishing, 2019. ISBN: 978-3-030-14022-9 978-3-030-14023-6. DOI: 10.1007/978-3-030-14023-6.

Langenbrunner, N. et al. « Dynamic Response of a Metal and a CMC Turbine Blade During a Controlled Rub Event Using a Segmented Shroud ». en. *Journal of Engineering for Gas Turbines and Power* 137.6 (June 2015). ISSN: 0742-4795. DOI: 10.1115/1.4028685.

Laverty, W. F. « Rub energetics of compressor blade tip seals ». en. *Wear* 75.1 (Jan. 1982), pages 1–20. ISSN: 0043-1648. DOI: 10.1016/0043-1648(82)90137-5.



Bibliographic references VI

Laxalde, D. et al. « Modelling and analysis of multi-stage systems of mistuned bladed disks ». en. *Computers & Structures* 89.3 (Feb. 2011), pages 316–324. ISSN: 0045-7949. DOI: 10.1016/j.compstruc.2010.10.020.

Laxalde, D. et al. « Nonlinear modal analysis of mechanical systems with frictionless contact interfaces ». en. *Computational Mechanics* 47.4 (Apr. 2011), pages 469–478. ISSN: 1432-0924. DOI: 10.1007/s00466-010-0556-3.

Lefcort, M. D. « An Investigation Into Unsteady Blade Forces in Turbomachines ». en. *Journal of Engineering for Power* 87.4 (Oct. 1965), pages 345–354. ISSN: 0022-0825. DOI: 10.1115/1.3678275.

Legrand, M. et al. « Full three-dimensional investigation of structural contact interactions in turbomachines ». en. *Journal of Sound and Vibration* 331.11 (May 2012), pages 2578–2601. ISSN: 0022-460X. DOI: 10.1016/j.jsv.2012.01.017.

Legrand, M. et al. « Numerical Investigation of Abradable Coating Removal in Aircraft Engines Through Plastic Constitutive Law ». en. *Journal of Computational and Nonlinear Dynamics* 7.1 (Jan. 2012). ISSN: 1555-1415. DOI: 10.1115/1.4004951.

Legrand, M. et al. « Numerical Investigation of Abradable Coating Wear Through Plastic Constitutive Law: Application to Aircraft Engines ». en. American Society of Mechanical Engineers Digital Collection, July 2010, pages 907–916. DOI: 10.1115/DETC2009-87669.

Legrand, M. et al. « Two-dimensional modeling of an aircraft engine structural bladed disk-casing modal interaction ». en. *Journal of Sound and Vibration* 319.1 (Jan. 2009), pages 366–391. ISSN: 0022-460X. DOI: 10.1016/j.jsv.2008.06.019.

Lesaffre, N. et al. « Contact analysis of a flexible bladed-rotor ». en. *European Journal of Mechanics - A/Solids* 26.3 (May 2007), pages 541–557. ISSN: 0997-7538. DOI: 10.1016/j.euromechsol.2006.11.002.

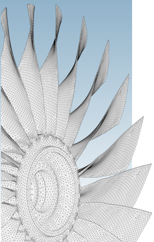
Ma, H. et al. « A review on dynamic characteristics of blade–casing rubbing ». en. *Nonlinear Dynamics* 84.2 (Apr. 2016), pages 437–472. ISSN: 1573-269X. DOI: 10.1007/s11071-015-2535-x.

Ma, H. et al. « A revised model for rubbing between rotating blade and elastic casing ». en. *Journal of Sound and Vibration* 337 (Feb. 2015), pages 301–320. ISSN: 0022-460X. DOI: 10.1016/j.jsv.2014.10.020.

Ma, H. et al. « Vibration response analysis of blade-disk dovetail structure under blade tip rubbing condition ». en. *Journal of Vibration and Control* 23.2 (2015), pages 252–271. DOI: 10.1177/1077546315575835.

Mailach, R. et al. « Rotor-Stator Interactions in a Four-Stage Low-Speed Axial Compressor—Part I: Unsteady Profile Pressures and the Effect of Clocking ». en. *Journal of Turbomachinery* 126.4 (Oct. 2004). Publisher: American Society of Mechanical Engineers Digital Collection, pages 507–518. ISSN: 0889-504X. DOI: 10.1115/1.1791641.

Mailach, R. et al. « Rotor-Stator Interactions in a Four-Stage Low-Speed Axial Compressor—Part II: Unsteady Aerodynamic Forces of Rotor and Stator Blades ». en. *Journal of Turbomachinery* 126.4 (Oct. 2004). Publisher: American Society of Mechanical Engineers Digital Collection, pages 519–526. ISSN: 0889-504X. DOI: 10.1115/1.1791642.



Bibliographic references VII

Mandard, R. et al. « Interacting force estimation during blade/seal rubs ». en. *Tribology International* 82 (Feb. 2015), pages 504–513. ISSN: 0301-679X. DOI: 10.1016/j.triboint.2014.01.026.

Maozhong, Y. et al. « Erosion wear behaviour and model of abradable seal coating ». en. *Wear* 252.1 (Jan. 2002), pages 9–15. ISSN: 0043-1648. DOI: 10.1016/S0043-1648(01)00681-0.

Marscher, W. D. « A phenomenological model of abradable wear in high performance turbomachinery ». en. *Wear* 59.1 (Mar. 1980), pages 191–211. ISSN: 0043-1648. DOI: 10.1016/0043-1648(80)90278-1.

Meingast, M. B. et al. « A linear complementarity problem formulation for periodic solutions to unilateral contact problems ». en. *International Journal of Non-Linear Mechanics*. 4th Canadian Conference on Non-linear Solid Mechanics 66 (Nov. 2014), pages 18–27. ISSN: 0020-7462. DOI: 10.1016/j.ijnonlinmec.2014.01.007.

Millecamps, A. et al. « Snecma's Viewpoint on the Numerical and Experimental Simulation of Blade-Tip/Casing Unilateral Contacts ». en. American Society of Mechanical Engineers Digital Collection, Aug. 2015. DOI: 10.1115/GT2015-42682.

Millecamps, A. et al. « Influence of Thermal Effects During Blade-Casing Contact Experiments ». en. American Society of Mechanical Engineers Digital Collection, July 2010, pages 855–862. DOI: 10.1115/DETC2009-86842.

Muszynska, A. « Rotor-to-stationary element sub-related vibration phenomena in rotating machinery: literature survey ». *Rotor-to-stationary element sub-related vibration phenomena in rotating machinery: literature survey* 21.3 (1989). Place: Thousands Oaks, CA Publisher: Sage, pages 3–11. ISSN: 0583-1024.

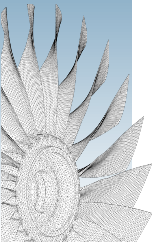
— .« Whirl and whip—Rotor/bearing stability problems ». en. *Journal of Sound and Vibration* 110.3 (Nov. 1986), pages 443–462. ISSN: 0022-460X. DOI: 10.1016/S0022-460X(86)80146-8.

Nitschke, S. et al. « An advanced experimental method and test rig concept for investigating the dynamic blade-tip/casing interactions under engine-like mechanical conditions ». en. *Wear* 422-423 (Mar. 2019), pages 161–166. ISSN: 0043-1648. DOI: 10.1016/j.wear.2018.12.072.

Novinski, E. et al. « Modified zirconia abradable seal coating for high temperature gas turbine applications ». en. *Thin Solid Films* 95.3 (Sept. 1982), pages 255–263. ISSN: 0040-6090. DOI: 10.1016/0040-6090(82)90018-9.

Nyssen, F. et al. « Experimental and numerical characterization of a ceramic matrix composite shroud segment under impact loading ». en. *Journal of Sound and Vibration* 467 (Feb. 2020), page 115040. ISSN: 0022-460X. DOI: 10.1016/j.jsv.2019.115040.

Nyssen, F. et al. « Thermo-Mechanical Modeling of Abradable Coating Wear in Aircraft Engines ». en. *Journal of Engineering for Gas Turbines and Power* 141.2 (Feb. 2019). ISSN: 0742-4795. DOI: 10.1115/1.4041647.



Bibliographic references VIII

Padova, C. et al. « Casing Treatment and Blade-Tip Configuration Effects on Controlled Gas Turbine Blade Tip/Shroud Rubs at Engine Conditions ». en. *Journal of Turbomachinery* 133.1 (Jan. 2011). ISSN: 0889-504X. DOI: 10.1115/1.4000539.

Padovan, J. et al. « Nonlinear Dynamics of Rotor/Blade/Casing Rub Interactions ». en. *Journal of Turbomachinery* 109.4 (Oct. 1987). Publisher: American Society of Mechanical Engineers Digital Collection, pages 527–534. ISSN: 0889-504X. DOI: 10.1115/1.3262143.

Paltrinieri, J. et al. « Towards Full 3D Numerical Simulation of Whirl Motions Stemming From Unilateral Contact Constraints in Aircraft Engines ». en. American Society of Mechanical Engineers Digital Collection, Aug. 2017. DOI: 10.1115/GT2017-63488.

Panunzio, A. M. et al. « Asymptotic Numerical Method and Polynomial Chaos Expansion for the Study of Stochastic Non-Linear Normal Modes ». en. American Society of Mechanical Engineers Digital Collection, Aug. 2015. DOI: 10.1115/GT2015-43560.

Papeleux, L. *Metafor*. 2020, METAFOR website.

Parent, M.-O. et al. « Whole Engine Interaction in a Bladed Rotor-to-Stator Contact ». en. American Society of Mechanical Engineers Digital Collection, Sept. 2014. DOI: 10.1115/GT2014-25253.

Patnaik, S. et al. « Modeling Nonlinear Oscillators via Variable-Order Fractional Operators ». en. American Society of Mechanical Engineers Digital Collection, Nov. 2019. DOI: 10.1115/DETC2019-97944.

Peletan, L. et al. « Quasi-periodic harmonic balance method for rubbing self-induced vibrations in rotor–stator dynamics ». en. *Nonlinear Dynamics* 78.4 (Dec. 2014), pages 2501–2515. ISSN: 1573-269X. DOI: 10.1007/s11071-014-1606-8.

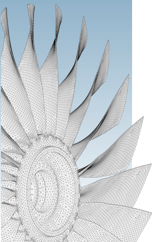
Petrov, E. P. « Analytical Formulation of Friction Contact Elements for Frequency-Domain Analysis of Nonlinear Vibrations of Structures With High-Energy Rubs ». en. *Journal of Engineering for Gas Turbines and Power* 141.12 (Dec. 2019). Publisher: American Society of Mechanical Engineers Digital Collection. ISSN: 0742-4795. DOI: 10.1115/1.4045183.

— .« Method for Direct Parametric Analysis of Nonlinear Forced Response of Bladed Discs With Friction Contact Interfaces ». en. American Society of Mechanical Engineers Digital Collection, Nov. 2008, pages 397–408. DOI: 10.1115/GT2004-53894.

— .« Multiharmonic Analysis of Nonlinear Whole Engine Dynamics With Bladed Disc-Casing Rubbing Contacts ». en. American Society of Mechanical Engineers Digital Collection, July 2013, pages 1181–1191. DOI: 10.1115/GT2012-68474.

Petrov, E. P. et al. « Generic Friction Models for Time-Domain Vibration Analysis of Bladed Disks ». en. *Journal of Turbomachinery* 126.1 (Jan. 2004). Publisher: American Society of Mechanical Engineers Digital Collection, pages 184–192. ISSN: 0889-504X. DOI: 10.1115/1.1644557.

Peyraut, F. et al. « Finite element modeling of abradable materials – Identification of plastic parameters and issues on minimum hardness against coating's thickness ». en. *Int. J. for Simulation and Multidisciplinary Design Optimization* 2.3 (July 2008), pages 209–215. ISSN: 1779-627X, 1779-6288. DOI: 10.1051/ijsmdo:2008028.



Bibliographic references IX

Piollet, E. et al. « Blade/casing rubbing interactions in aircraft engines: Numerical benchmark and design guidelines based on NASA rotor 37 ». en. *Journal of Sound and Vibration* 460 (Nov. 2019), page 114878. ISSN: 0022-460X. DOI: 10.1016/j.jsv.2019.114878.

Piollet, E. et al. *A program to compute compressor blade geometries from multiple-circular-arc parameters with sweep and lean (v1.0)*. May 2019, code available online.

Platzer, M. F. et al. *AGARD Manual on Aeroelasticity in Axial-Flow Turbomachines. Volume 1. Unsteady Turbomachinery aerodynamics*. 1987, available online.

Prabith, K. et al. « The numerical modeling of rotor–stator rubbing in rotating machinery: a comprehensive review ». en. *Nonlinear Dynamics* (July 2020). ISSN: 1573-269X. DOI: 10.1007/s11071-020-05832-y.

Rathmann, U. et al. « Sealing Technology: Rub Test Rig for Abrasive/Abradable Systems ». en. American Society of Mechanical Engineers Digital Collection, Mar. 2009, pages 223–228. DOI: 10.1115/GT2007-27724.

Salles, L. et al. « Dynamic Analysis of Fretting-Wear in Joint Interface by a Multiscale Harmonic Balance Method Coupled With Explicit or Implicit Integration Schemes ». en. American Society of Mechanical Engineers Digital Collection, Dec. 2010, pages 1003–1013. DOI: 10.1115/GT2010-23264.

Salles, L. et al. « Continuation techniques for analysis of whole aeroengine dynamics with imperfect bifurcations and isolated solutions ». en. *Nonlinear Dynamics* 86.3 (Nov. 2016), pages 1897–1911. ISSN: 1573-269X. DOI: 10.1007/s11071-016-3003-y.

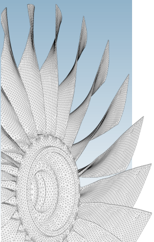
Salvat, N. et al. « Two-dimensional modeling of unilateral contact-induced shaft precessional motions in bladed-disk/casing systems ». en. *International Journal of Non-Linear Mechanics* 78 (Jan. 2016), pages 90–104. ISSN: 0020-7462. DOI: 10.1016/j.ijnonlinmec.2015.10.001.

Schmid, R. K. « New high temperature abradables for gas turbines ». en. PhD thesis. ETH Zurich, 1997. DOI: 10.3929/ethz-a-001809249.

Schmiechen, P. « Travelling Wave Speed Coincidence ». en. PhD thesis. 1997, available online.

Schmitz, M. B. et al. « Axial Turbine Blade Vibrations Induced by the Stator Flow ». en. *Unsteady Aerodynamics, Aeroacoustics and Aeroelasticity of Turbomachines*. Dordrecht: Springer Netherlands, 2006, pages 107–118. ISBN: 978-1-4020-4605-6. DOI: 10.1007/1-4020-4605-7_9.

Seinturier, E. et al. « Turbine Mistuned Forced Response Prediction: Comparison With Experimental Results ». en. American Society of Mechanical Engineers Digital Collection, Feb. 2009, pages 943–952. DOI: 10.1115/GT2002-30424.



Bibliographic references X

Shiembob, L. T. *Continued development of abradable gas path seals. [for gas turbine engines]*. Technical report. https://ntrs.nasa.gov/search.jsp?R=19760004187. Nov. 1975, available online.

Simon Chong Escobar, A. et al. « Path-Following Bifurcation Analysis of Church Bell Dynamics ». en. *Journal of Computational and Nonlinear Dynamics* 12.6 (Nov. 2017). Publisher: American Society of Mechanical Engineers Digital Collection. ISSN: 1555-1415. DOI: 10.1115/1.4036114.

Sinha, S. K. « Non-linear dynamic response of a rotating radial Timoshenko beam with periodic pulse loading at the free-end ». en. *International Journal of Non-Linear Mechanics* 40.1 (Jan. 2005), pages 113–149. ISSN: 0020-7462. DOI: 10.1016/j.ijnonlinmec.2004.05.019.

Skiba, S. et al. « Experimental Investigation of the Mechanical Behavior of an AlSi-PE Abradable Coating at High Strain Rates for a Large Range of Temperatures ». en. *Journal of Dynamic Behavior of Materials* (Mar. 2020). ISSN: 2199-7454. DOI: 10.1007/s40870-020-00242-y.

Sternchuss, A. et al. « On the reduction of quasi-cyclic disk models with variable rotation speeds ». *International conference on noise and vibration engineering*. Louvain, 2006, available online. ISBN: 90-73802-83-0.

Thorin, A. et al. « Nonsmooth Modal Analysis of Piecewise-Linear Impact Oscillators ». *SIAM Journal on Applied Dynamical Systems* 16.3 (Jan. 2017). Publisher: Society for Industrial and Applied Mathematics, pages 1710–1747. DOI: 10.1137/16M1081506.

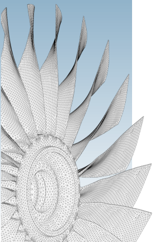
Thorin, A. et al. « Nonsmooth Thermoelastic Simulations of Blade–Casing Contact Interactions ». en. *Journal of Engineering for Gas Turbines and Power* 141.2 (Feb. 2019). ISSN: 0742-4795. DOI: 10.1115/1.4040857.

Thouverez, F. « PHARE Plateforme macHines tournantes pour la mAîtrise des Risques Environnementaux ». fr. *PHARE Plateforme macHines tournantes pour la mAîtrise des Risques Environnementaux* (), page 5. DOI: https://www.ec-lyon.fr/campus/vie-campus/grands-projets/equipex-phare.

Tran, D.-M. « Reduced models of multi-stage cyclic structures using cyclic symmetry reduction and component mode synthesis ». en. *Journal of Sound and Vibration* 333.21 (Oct. 2014), pages 5443–5463. ISSN: 0022-460X. DOI: 10.1016/j.jsv.2014.06.004.

Turner, K. E. et al. « Airfoil Deflection Characteristics During Rub Events ». en. American Society of Mechanical Engineers Digital Collection, Dec. 2010, pages 779–789. DOI: 10.1115/GT2010-22166.

Verdon, J. M. et al. « Development of a Linear Unsteady Aerodynamic Analysis for Finite-Deflection Subsonic Cascades ». *AIAA Journal* 20.9 (1982). Publisher: American Institute of Aeronautics and Astronautics _eprint: https://doi.org/10.2514/3.51186, pages 1259–1267. ISSN: 0001-1452. DOI: 10.2514/3.51186.



Bibliographic references XI

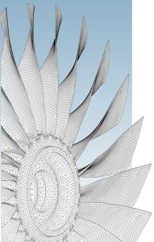
Williams, R. J. « Simulation of Blade Casing Interaction Phenomena in Gas Turbines Resulting From Heavy Tip Rubs Using an Implicit Time Marching Method ». en. American Society of Mechanical Engineers Digital Collection, May 2012, pages 1007–1016. DOI: 10.1115/GT2011-45495.

Wriggers, P. *Computational Contact Mechanics*. en. 2nd edition. Berlin Heidelberg: Springer-Verlag, 2006. ISBN: 978-3-540-32608-3. DOI: 10.1007/978-3-540-32609-0.

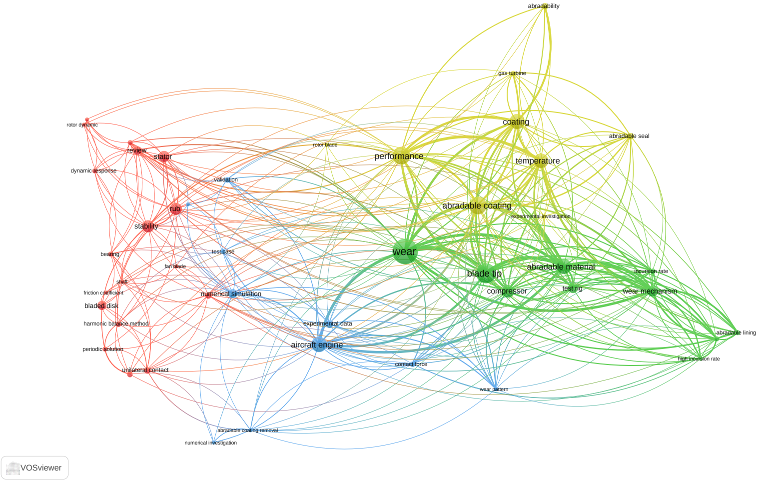
Yi, M. et al. « Abradability evaluation and tribological behaviour of abradable seal coating ». 8.3 (1998, available online), pages 459–467.

Yoong, C. et al. « Nonsmooth modal analysis of an elastic bar subject to a unilateral contact constraint ». en. *Nonlinear Dynamics* 91.4 (Mar. 2018), pages 2453–2476. ISSN: 1573-269X. DOI: 10.1007/s11071-017-4025-9.

Zhang, B. et al. « Investigating material removal mechanism of Al-Si base abradable coating in labyrinth seal system ». en. *Wear*. 22nd International Conference on Wear of Materials 426-427 (Apr. 2019), pages 239–249. ISSN: 0043-1648. DOI: 10.1016/j.wear.2019.01.034.



Bibliographic references XII



- Visual representation of the bibliographic network of keywords of the referenced publications (obtained with the VOSviewer software)